Lectures on quantum gases

Lecture 5

Cold Collisions

Feshbach resonances

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Lecture notes:
https://staff.fnwi.uva.nl/j.t.m.walraven/walraven/JookWalraven.htm

Interactions between two alkali atoms

Hamiltonian including spin Zeeman term:

$$\mathcal{V}(r) = V_D(r) + J(r)\mathbf{s}_1 \cdot \mathbf{s}_2$$

$$\mathcal{H} = \frac{1}{2\mu}\left(p_r^2 + \frac{\mathbf{L}^2}{r^2}\right) + \mathcal{V}(r) + \gamma_e B\, S_z \underbrace{- (\gamma_1 i_{z1} + \gamma_2 i_{z2})\, B}_{(-\gamma_n B\, I_z)} + \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$

all terms conserve M_F

only term *not* singlet/triplet conserving

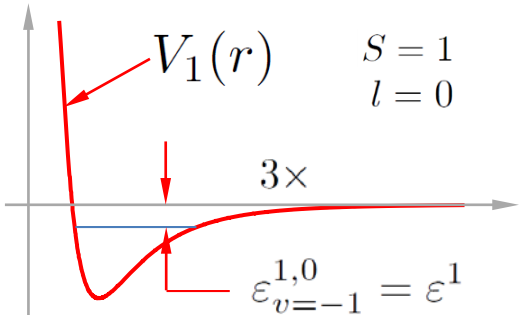
Good basis states: $|\psi\rangle = |R_l\rangle|lm_l;\psi_e\rangle\,|S,M_S\rangle\,|i_1 i_2 m_1 m_2\rangle$

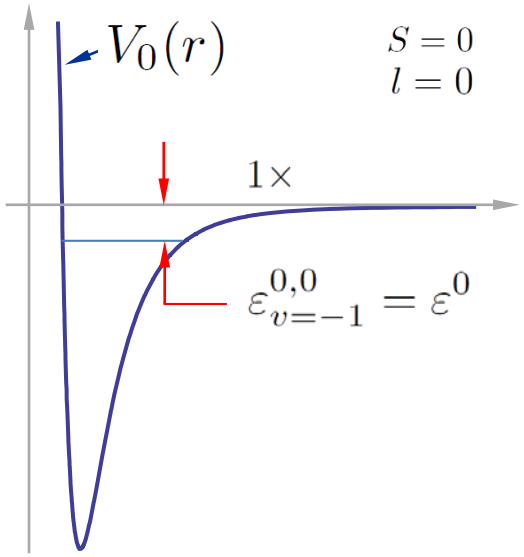
$$|\psi\rangle = |R_l\rangle|lm_l;\psi_e\rangle\,|S,M_S\rangle\,|IM_I\rangle$$

Effective potential:

$$U_{S,l}(r) = U_S(r) + \frac{l\,(l+1)}{r^2} + \frac{2\mu}{\hbar^2}\left[\gamma_e B\, M_S \underbrace{- (\gamma_1 m_1 + \gamma_2 m_2)\, B}_{(-\gamma_n B M_I)}\right]$$

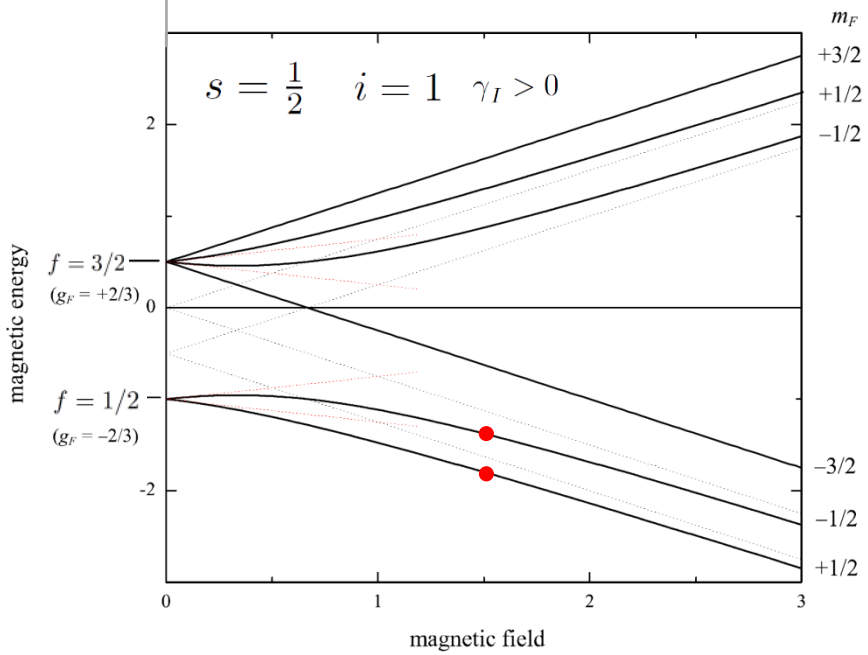
Example: two ^{6}Li atoms





$\gamma_I > 0$

$s = \frac{1}{2} \quad i = 1 \rightarrow f = \frac{1}{2}, \frac{3}{2}$



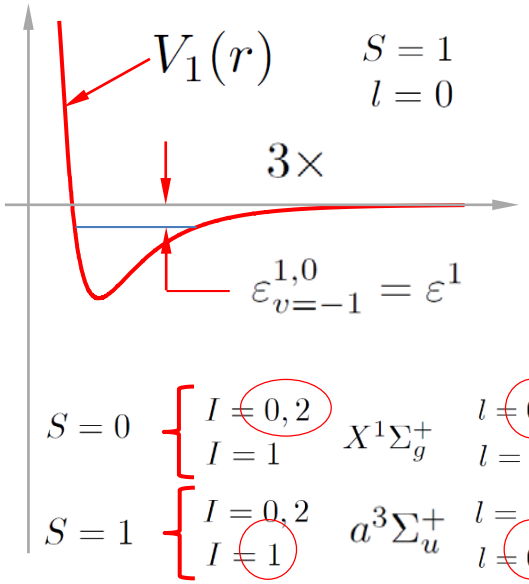
Find all s-wave molecules with $M_F = 0$

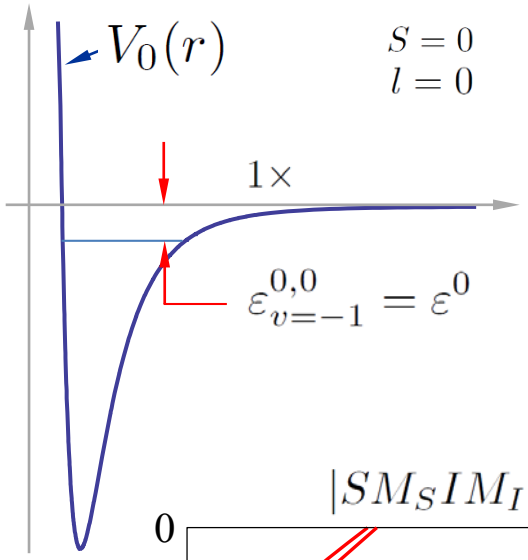
$$S = 0 \begin{cases} I = 0, 2 \\ I = 1 \end{cases} X^1\Sigma_g^+ \quad \begin{matrix} l = 0, 2, 4, \cdots \\ l = 1, 3, 5, \cdots \end{matrix}$$

$$S = 1 \begin{cases} I = 0, 2 \\ I = 1 \end{cases} a^3\Sigma_u^+ \quad \begin{matrix} l = 1, 3, 5, \cdots \\ l = 0, 2, 4, \cdots \end{matrix}$$

→ free atom pair with $M_F = 0$

Example: two ^{6}Li atoms

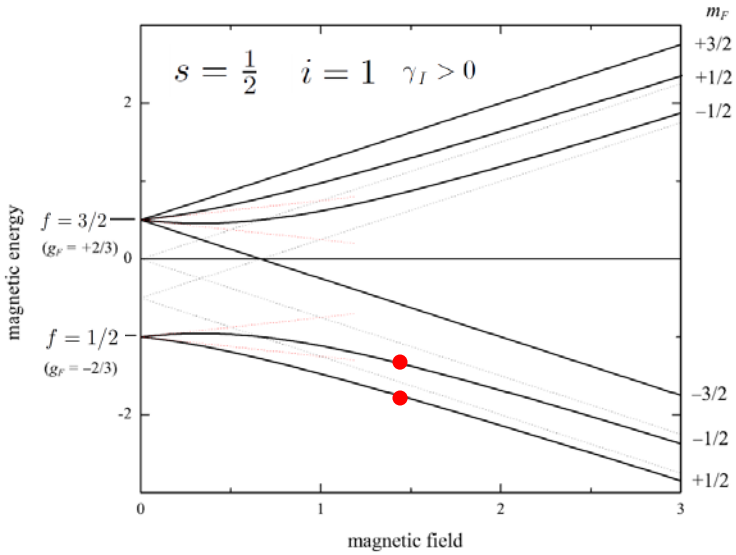


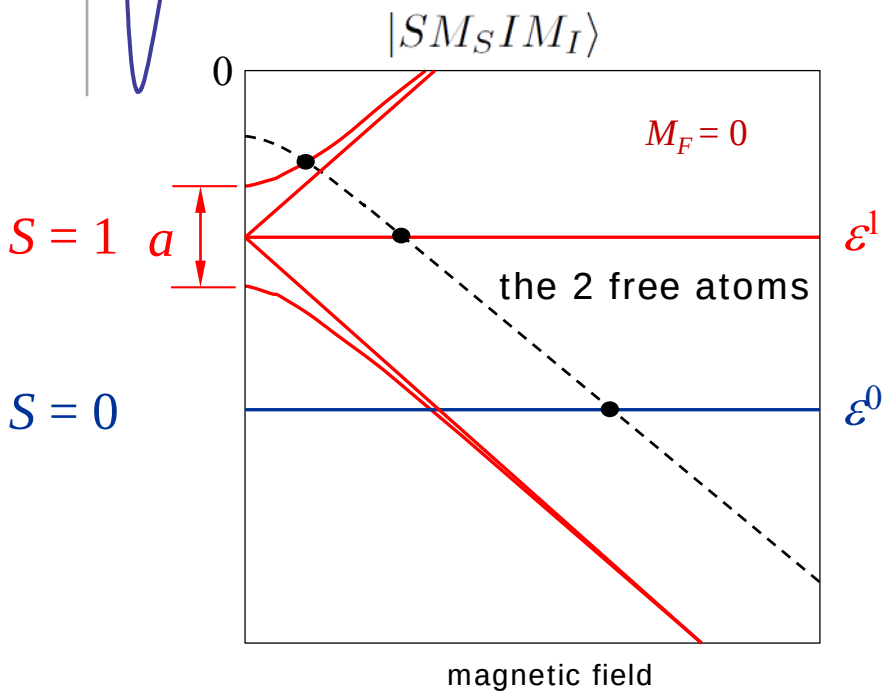


$M_F = 0$

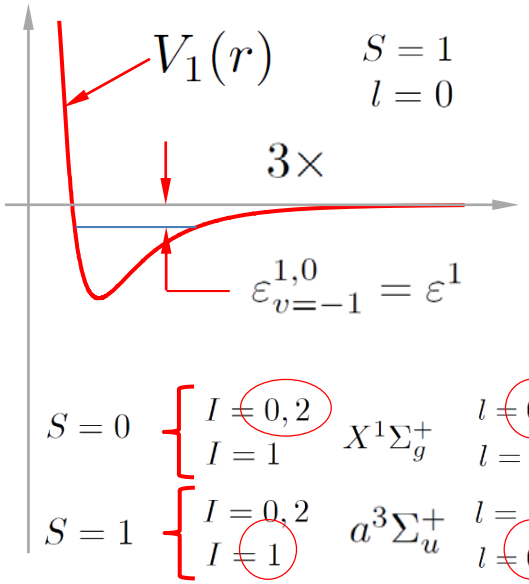
$S = 0$ { $I = 0, 2$; $I = 1$ } $X^1\Sigma_g^+$ { $l = 0, 2, 4, \cdots$; $l = 1, 3, 5, \cdots$ }

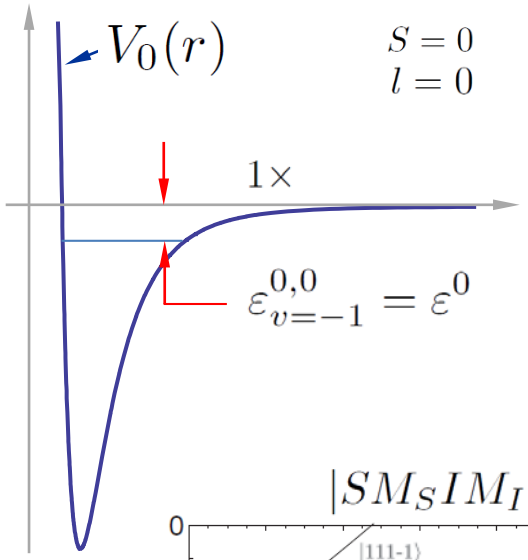
$S = 1$ { $I = 0, 2$; $I = 1$ } $a^3\Sigma_u^+$ { $l = 1, 3, 5, \cdots$; $l = 0, 2, 4, \cdots$ }





Example: two ^{6}Li atoms

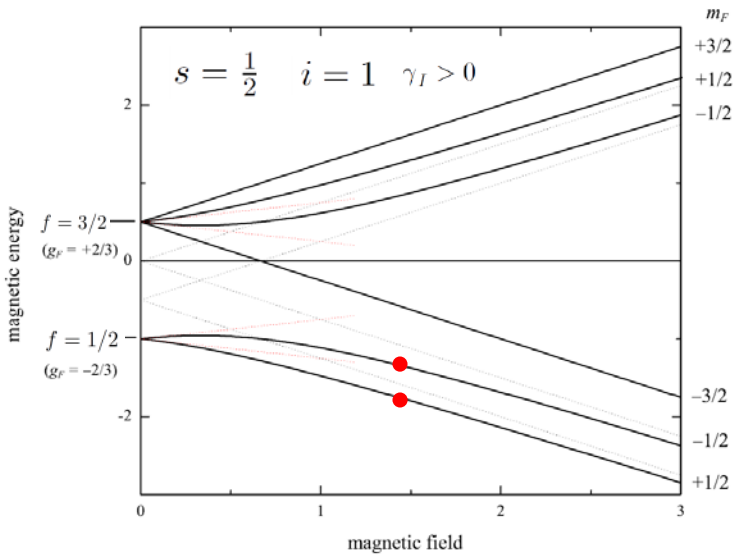




$M_F = 0$

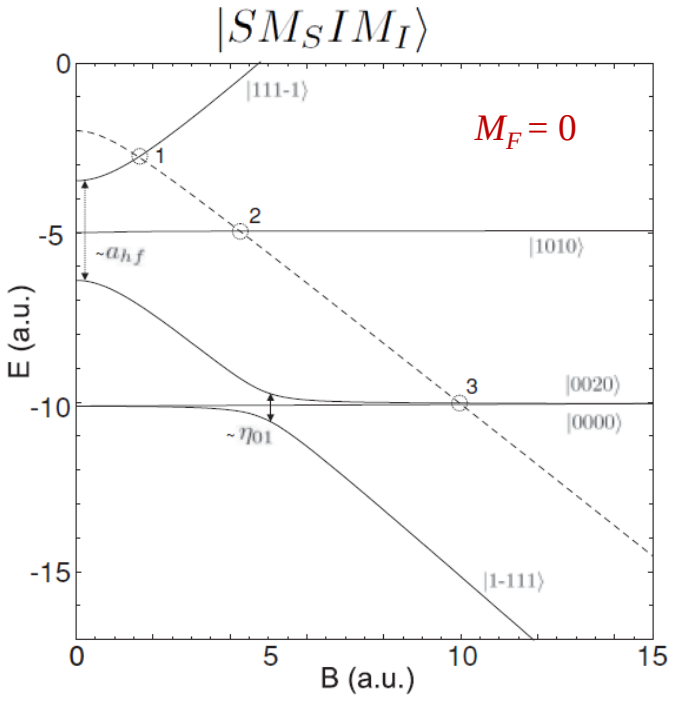
$S = 0$ { $I = 0, 2$ / $I = 1$ } $X^1\Sigma_g^+$ { $l = 0, 2, 4, \cdots$ / $l = 1, 3, 5, \cdots$ }

$S = 1$ { $I = 0, 2$ / $I = 1$ } $a^3\Sigma_u^+$ { $l = 1, 3, 5, \cdots$ / $l = 0, 2, 4, \cdots$ }



$S = 1$

$S = 0$



Diagonalization of Hamiltonian

$$\mathcal{H} = \frac{1}{2\mu}\left(p_r^2 + \frac{\mathbf{L}^2}{r^2}\right) + \mathcal{V}(r) + \gamma_e B\, S_z - (\gamma_1 i_{z1} + \gamma_2 i_{z2})\, B + \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$

$$\det\left|\langle\beta|\mathcal{H}|\alpha\rangle - \underline{I}E\right| = 0$$

$$|\alpha\rangle = |R_{l,v}^{S}\rangle \overbrace{|S, M_S\rangle|i_1, m_1\rangle|i_2, m_2\rangle}^{|\sigma\rangle}$$

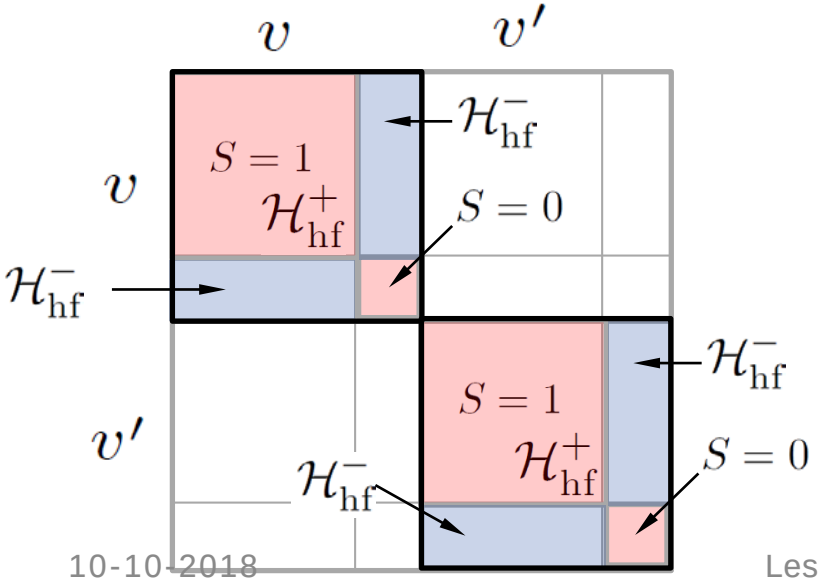
$$\langle\beta|\mathcal{H}|\alpha\rangle = \left[\varepsilon_v^{S,l} + (\gamma_e M_S - \gamma_1 m_1 + \gamma_2 m_2)\, B\right]\langle\beta|\alpha\rangle + \langle\beta|\mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}|\alpha\rangle$$

$$= \varepsilon_v^{S,l}(B)\delta_{v,v'}\delta_{\sigma,\sigma'} + \underbrace{\langle\sigma'|\mathcal{H}_{\rm hf}^{+}|\sigma\rangle}_{=0}\delta_{v,v'} + \underbrace{\langle\sigma'|\mathcal{H}_{\rm hf}^{-}|\sigma\rangle}_{=0}\langle R_{v'}^{S',l}|R_v^{S,l}\rangle$$

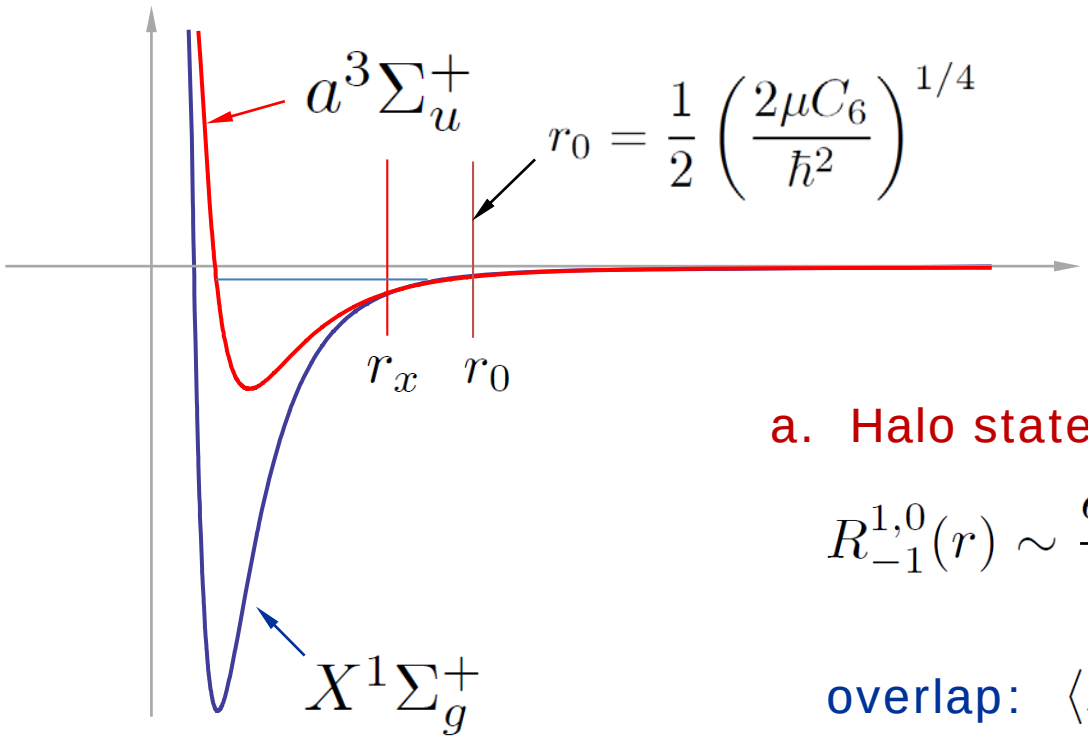
$=0$ for $S \neq S'$

$=0$ for $S = S'$

Franck-Condon factor (only part that depends on radial wavefunction)



Franck-Condon factor



a. Halo states (universal regime)

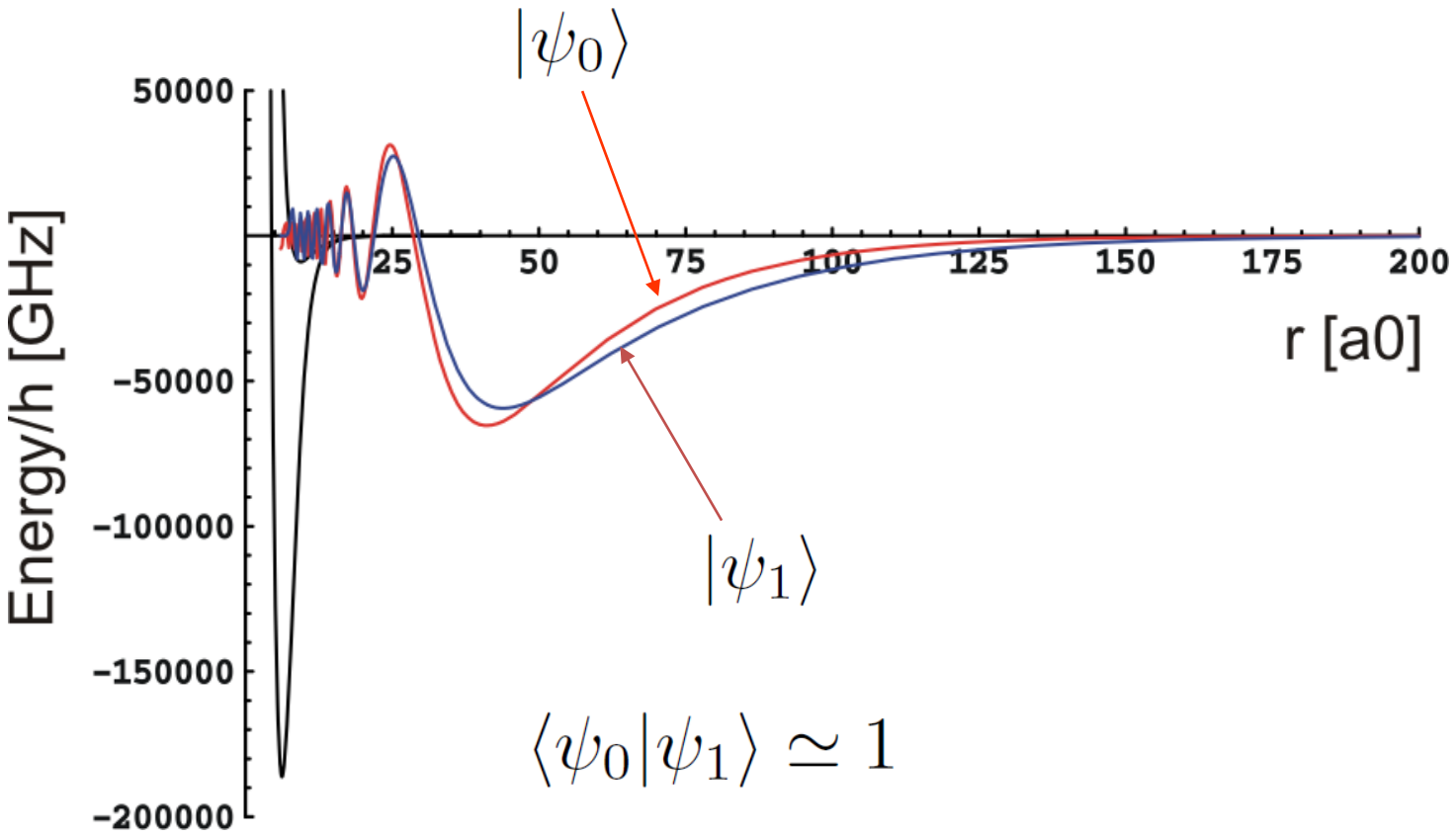
$$R_{-1}^{1,0}(r) \sim \frac{e^{-\kappa_1 r}}{r} \qquad R_{-1}^{0,0}(r) \sim \frac{e^{-\kappa_0 r}}{r}$$

overlap: $\langle R_{-1}^{0,0} | R_{-1}^{1,0} \rangle = 2\frac{\sqrt{\kappa_0 \kappa_1}}{\kappa_0 + \kappa_1}$

b. Asymptotic bound states ($r_{cl} > r_x$)

can be calculated numerically
starting from Van der Waals tail

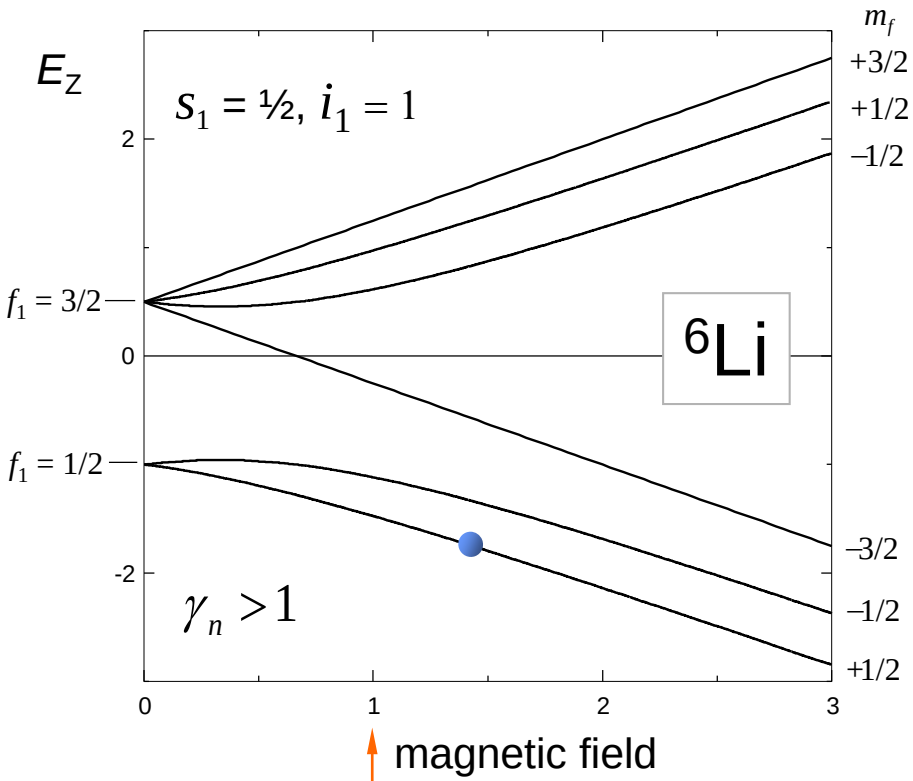
singlet/triplet overlap

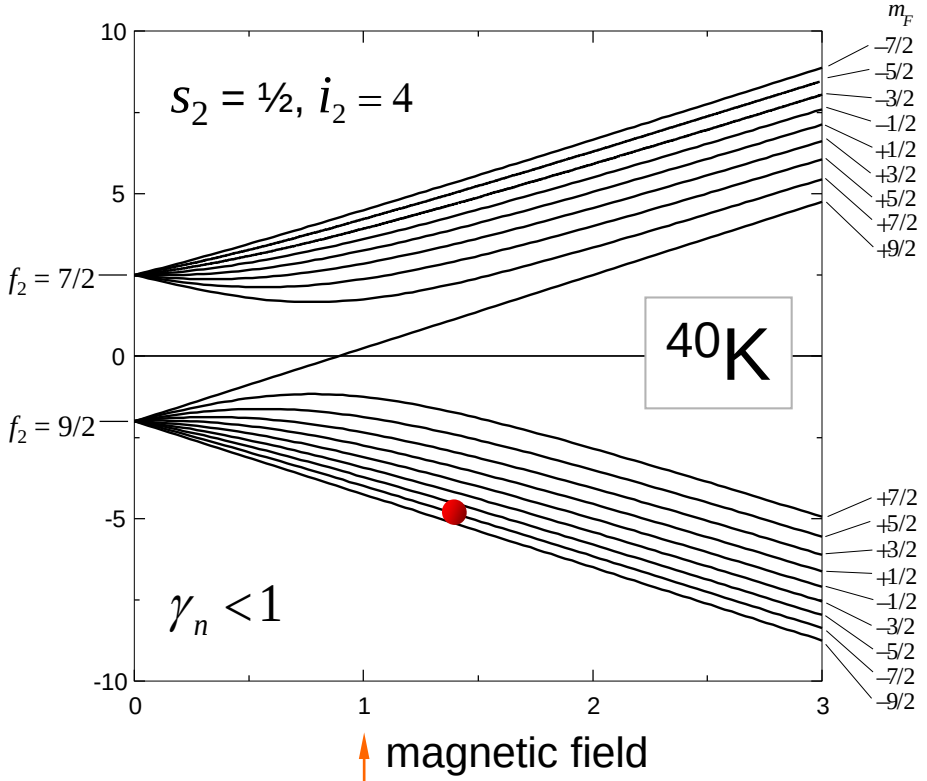


Potentials reduce to two parameters: ϵ_0 ϵ_1

^{6}Li-^{40}K pairs

$$\mathcal{H}_{\mathrm{hf+Z}} = \frac{a_1}{\hbar^2}\mathbf{i}_1 \cdot \mathbf{s}_1 + \frac{a_2}{\hbar^2}\mathbf{i}_2 \cdot \mathbf{s}_2 + \gamma_s \left(\mathbf{s}_1 + \mathbf{s}_2\right) \cdot \mathbf{B} - \left(\gamma_1 \mathbf{i}_1 + \gamma_2 \mathbf{i}_2\right) \cdot \mathbf{B}$$

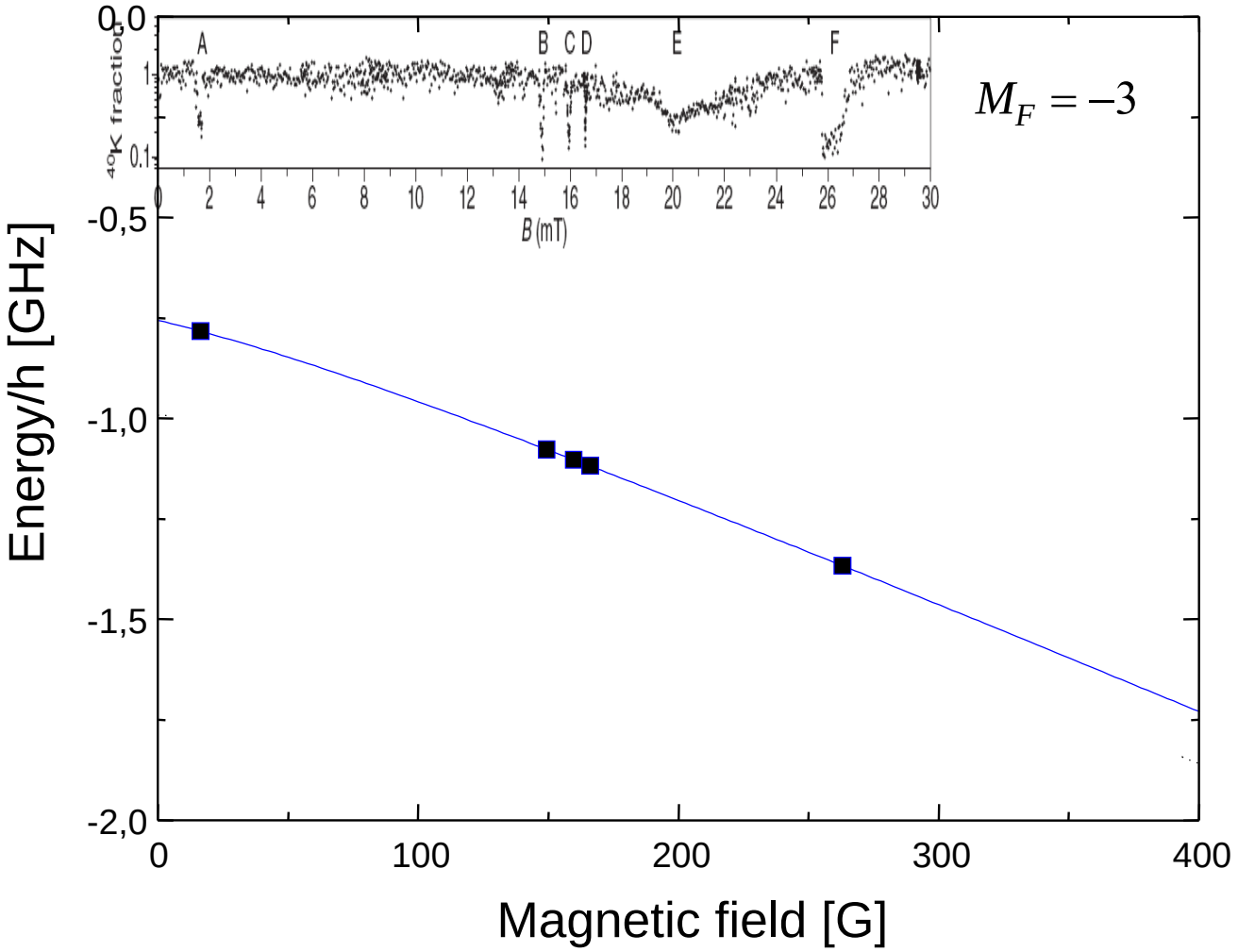




$M_F = -3$

^{6}Li-^{40}K loss features with $M_F = -3$

$$\left|\frac{9}{2}, -\frac{7}{2}\right\rangle_{\mathrm{K}} + \left|\frac{1}{2}, \frac{1}{2}\right\rangle_{\mathrm{Li}}$$



^{6}Li-^{40}K loss features with $M_F = -3$

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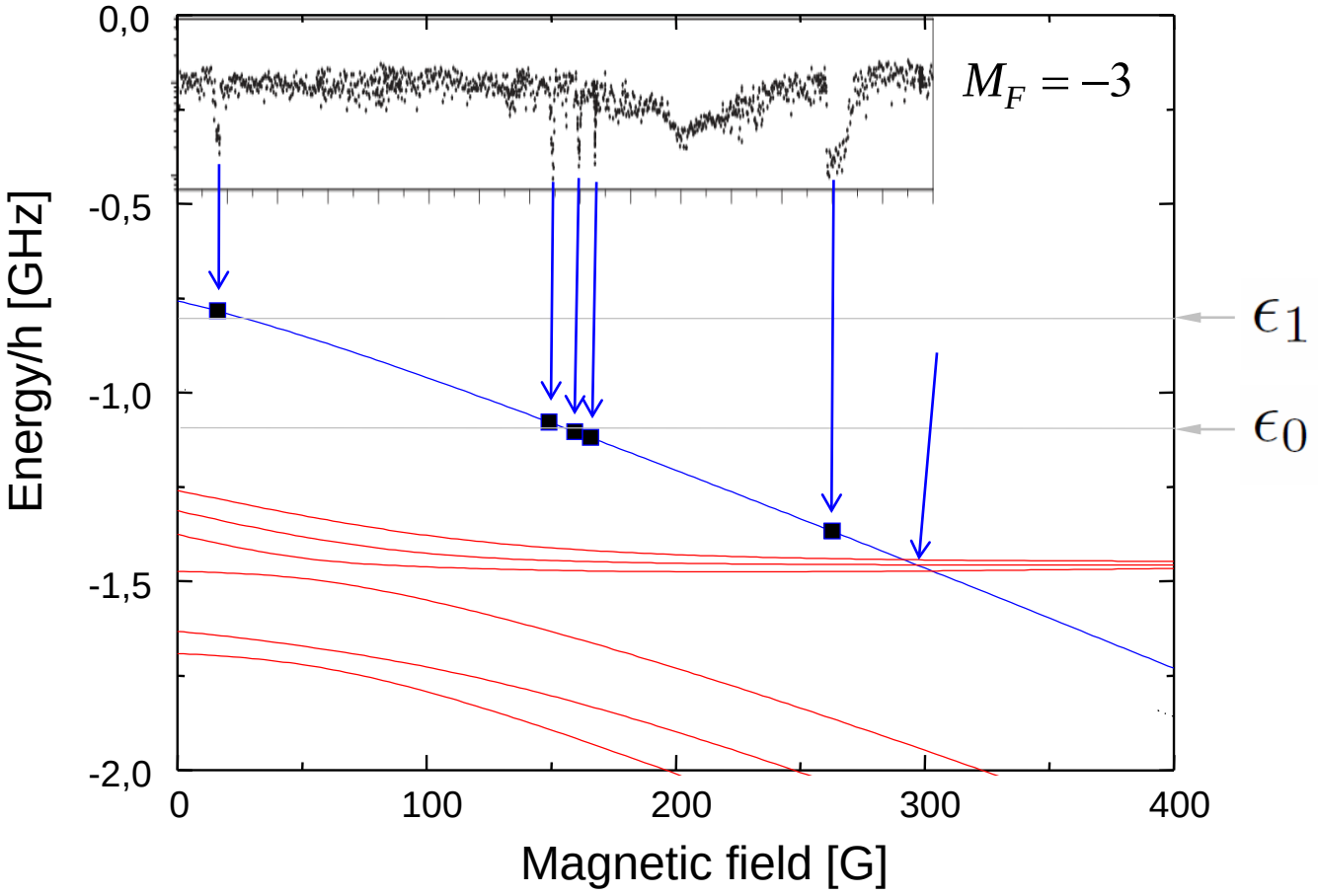
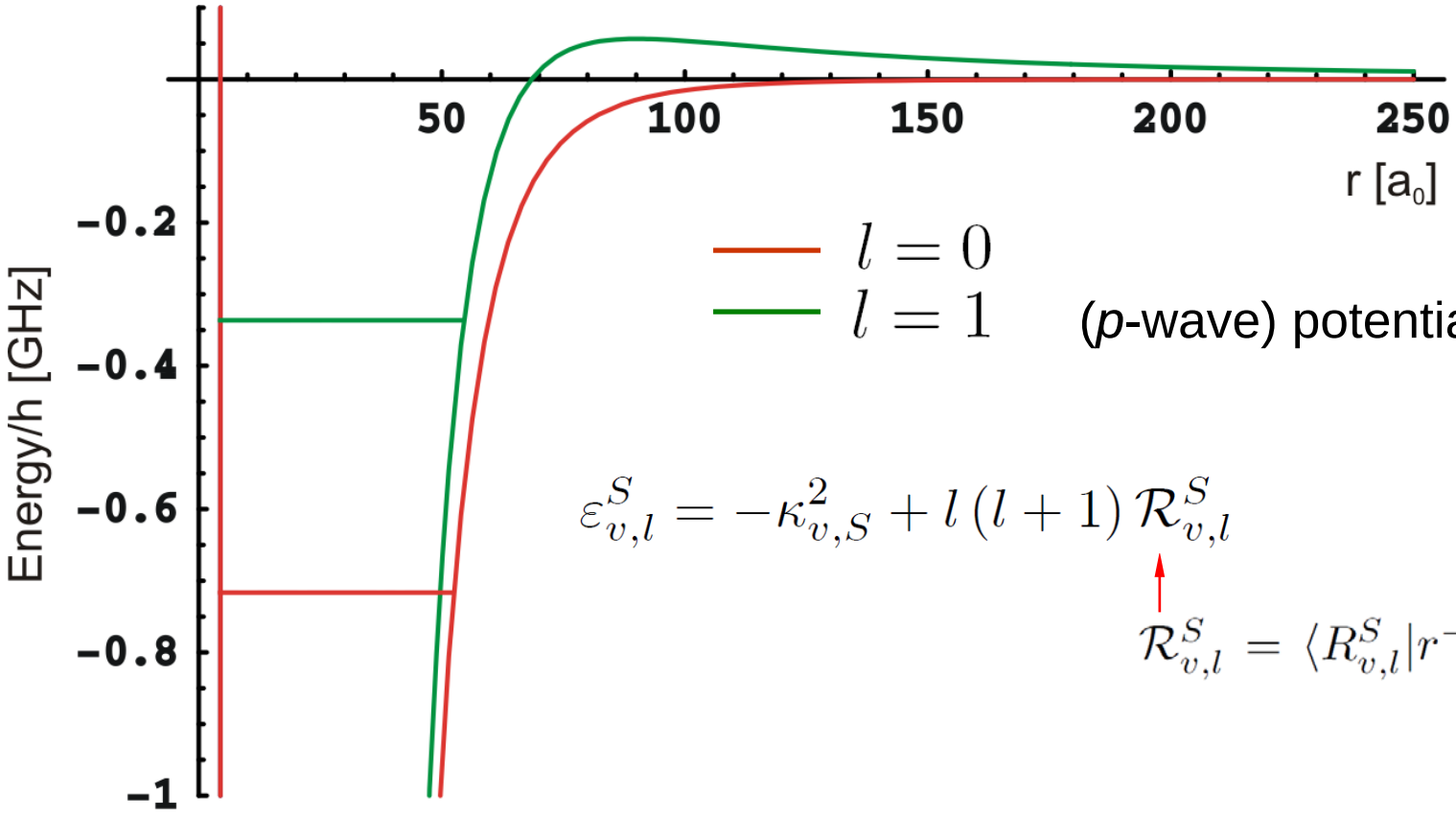


Include $l = 1$ molecular levels



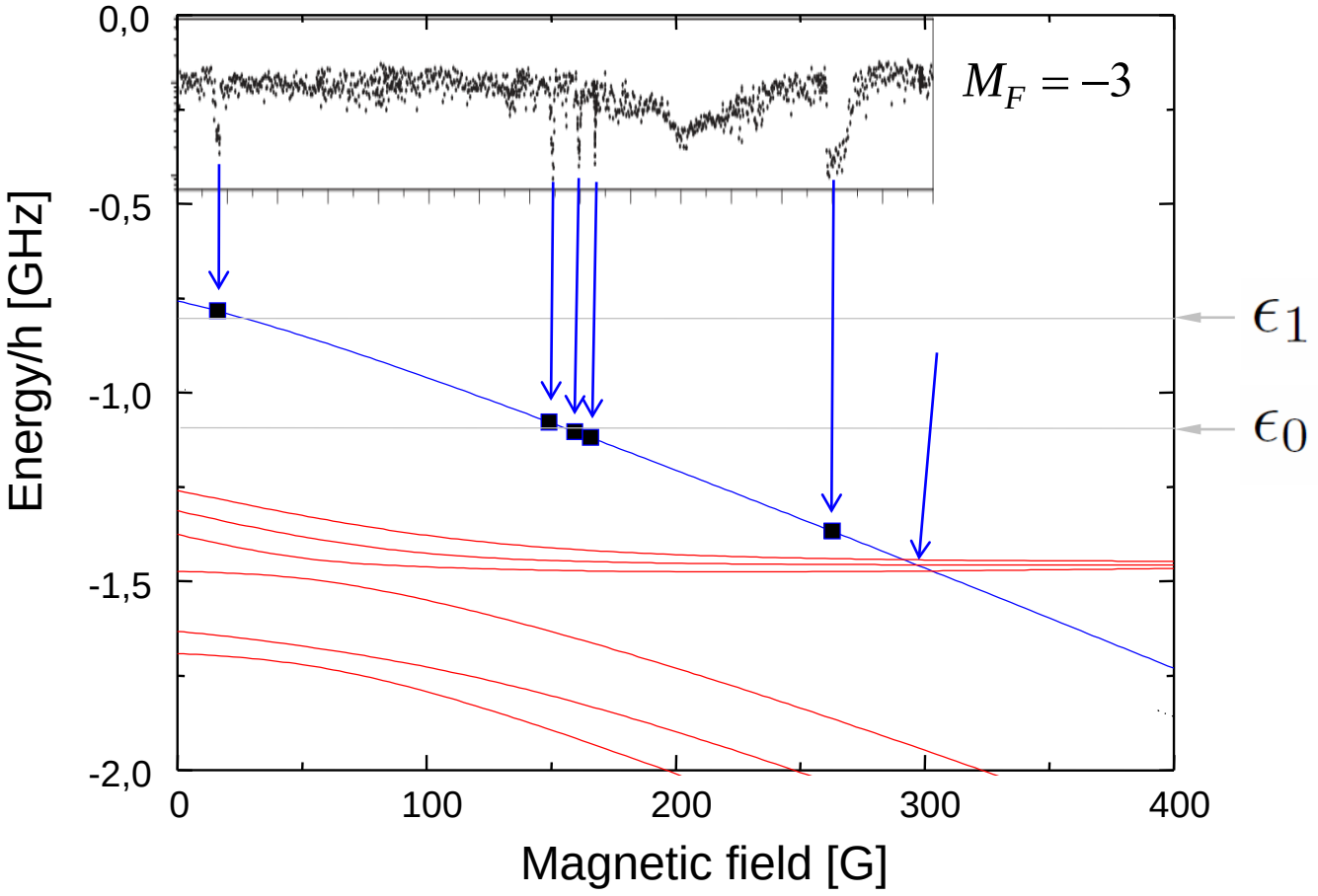
$$\varepsilon^S_{v,l} = -\kappa^2_{v,S} + l\,(l+1)\,\mathcal{R}^S_{v,l}$$

$$\mathcal{R}^S_{v,l} = \langle R^S_{v,l} | r^{-2} | R^S_{v,l} \rangle$$

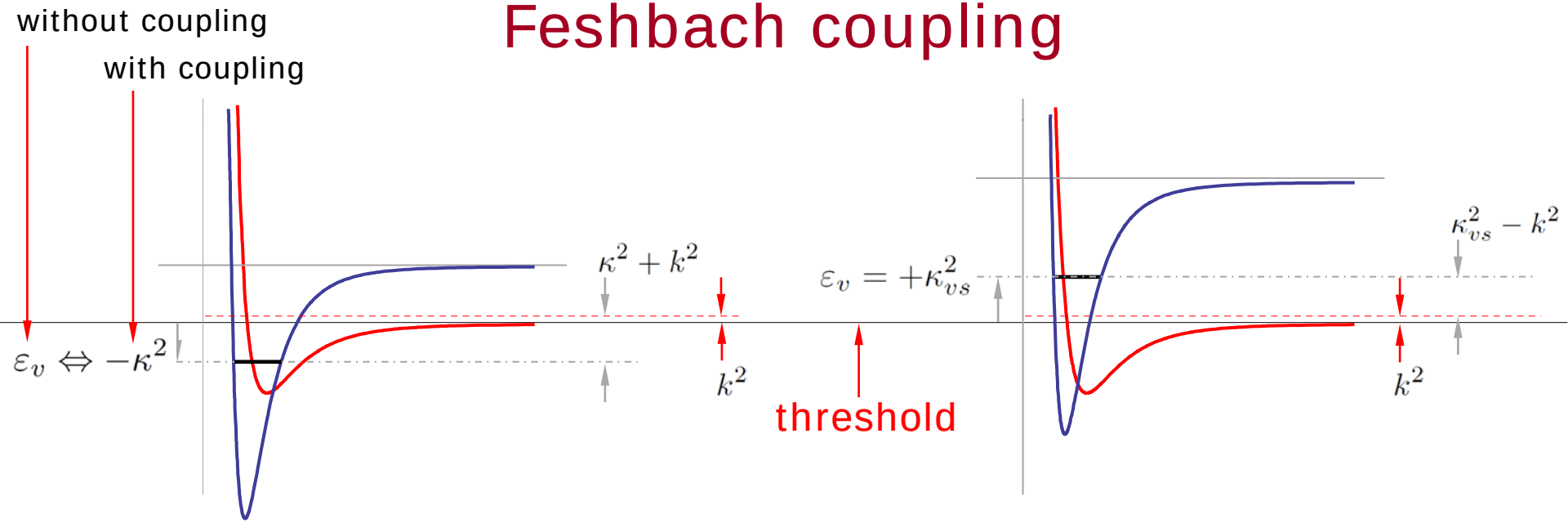
Conclusion: $l = 0$ and $l = 1$ levels can be related through C_6 coefficient

^{6}Li-^{40}K loss features with $M_F = -3$

$$|\tfrac{9}{2}, -\tfrac{7}{2}\rangle_{\mathrm{K}} + |\tfrac{1}{2}, \tfrac{1}{2}\rangle_{\mathrm{Li}}$$

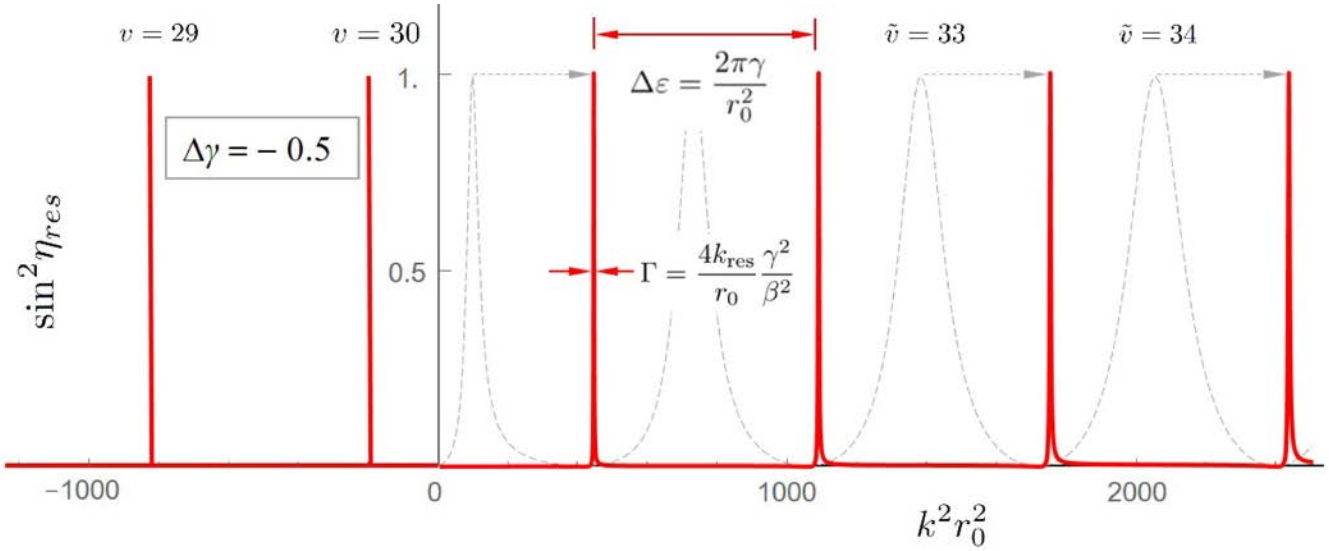


Feshbach coupling



We ask for the relation between ε_v and $-\kappa^2$

screened flat-bottom potential

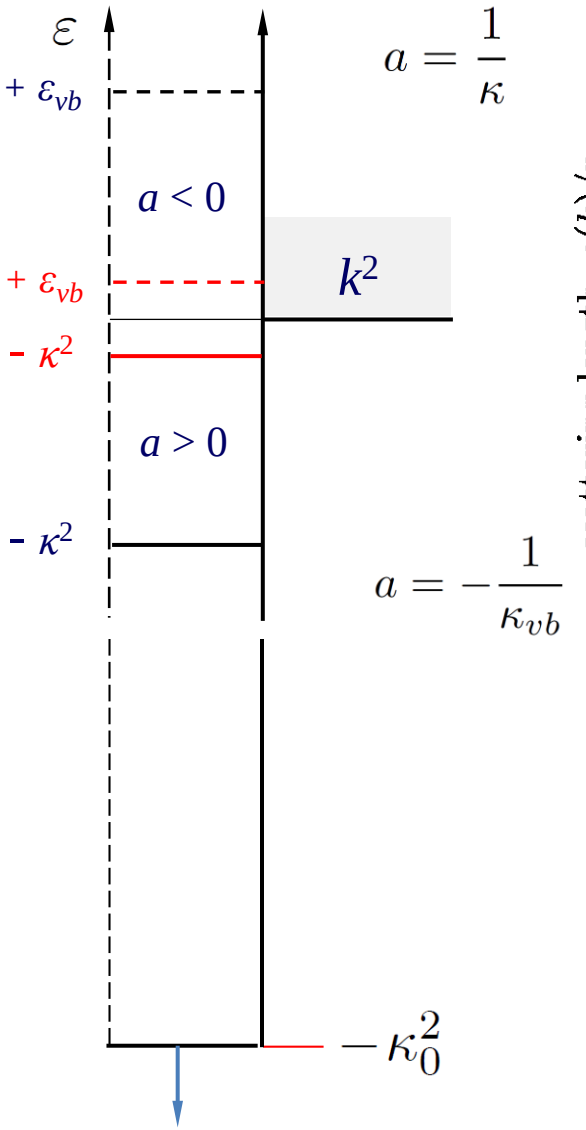


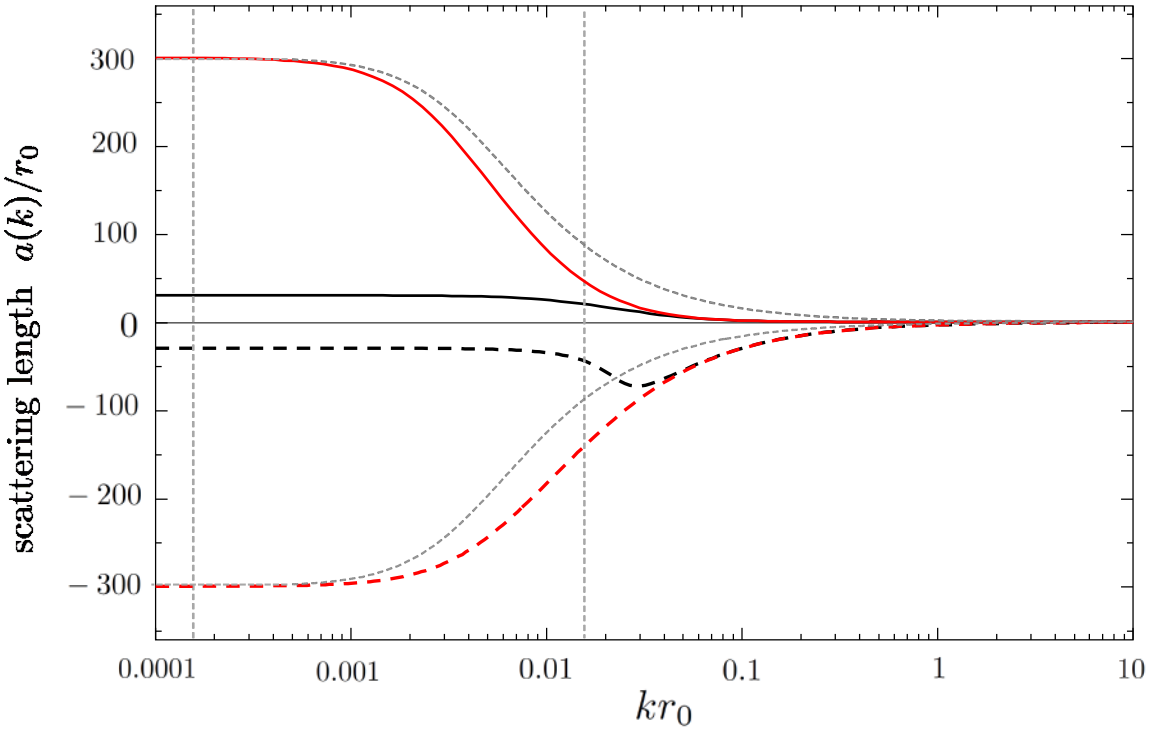
New notation: expand about resonance ($k \simeq k_{\rm res}$)

$$\tan\eta_{\rm res} = \frac{k}{K_+\cot K_+ r_0 - \kappa_1} \simeq \frac{-\Gamma/2}{\varepsilon - \varepsilon_{\rm res}} \qquad \Gamma/2 = (2k_{\rm res}/r_0)\frac{\gamma^2}{\beta^2} = k_{res}/R^*$$

$$\sin^2\eta_{\rm res} = \frac{(\Gamma/2)^2}{(\varepsilon - \varepsilon_{\rm res})^2 + (\Gamma/2)^2}$$

narrow resonance near threshold





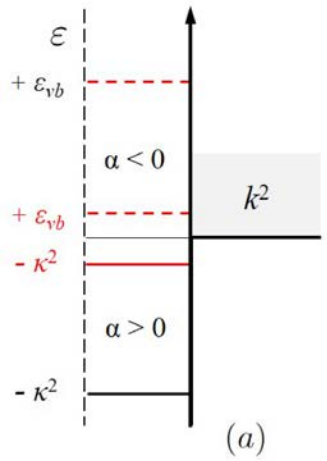
$$a(k) = r_0 - \frac{1}{k}\arctan\left(\frac{kr_0}{K_+ r_0 \cot K_+ r_0 - \beta}\right)$$

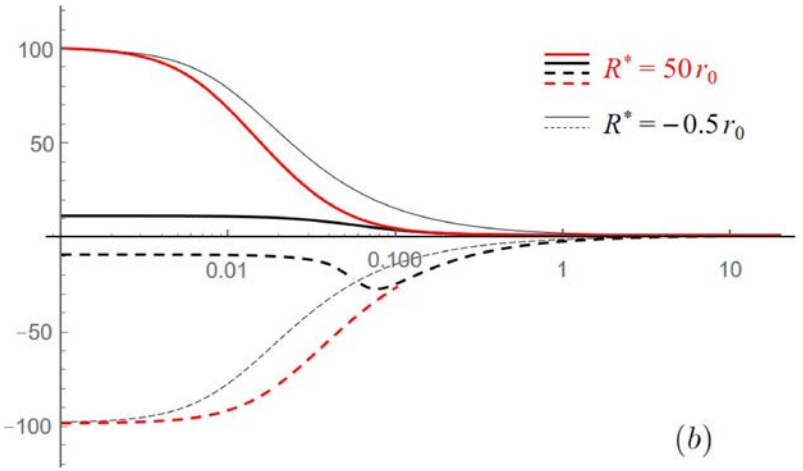
$$K_+^2 = k^2 + \kappa_0^2$$

resonance near threshold

$$a = \lim_{k\to 0} a(k)$$

$$k\cot\eta_{res} = -\frac{1}{a_{res}(k)}$$





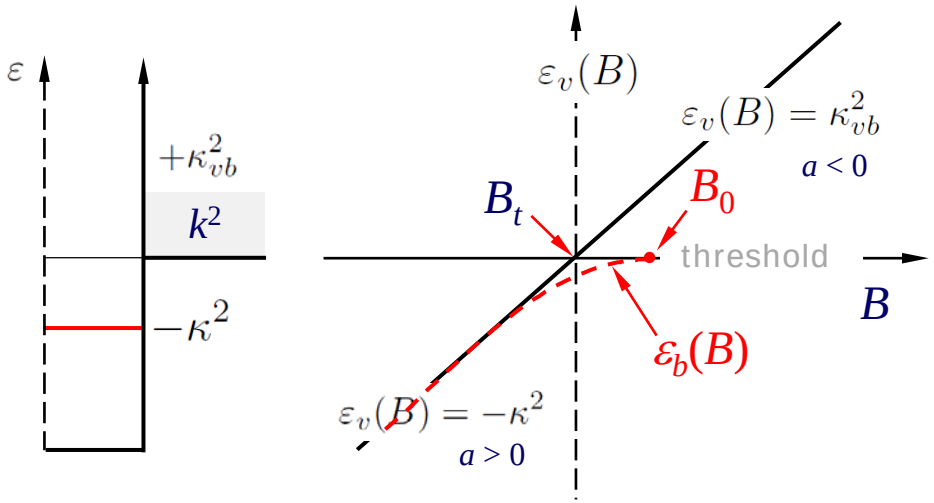
weakly bound level: $k\cot\eta_{res} = -\kappa - (k^2+\kappa^2)R^* \qquad \Gamma/2 = \kappa/R^*$

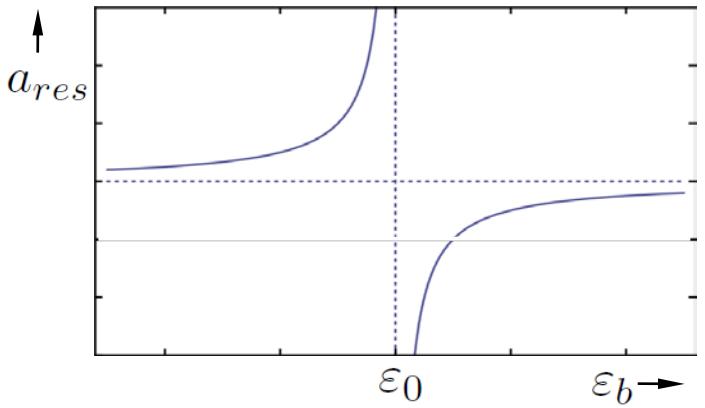
$$a_{res}(k) = \frac{1}{\kappa + (k^2+\kappa^2)R^*} \xrightarrow[k\to 0]{} \overbrace{\frac{1}{\kappa+\kappa^2 R^*}}^{a_{res}} \xrightarrow[\kappa\to 0]{} \frac{1}{\kappa}$$

virtual level: $k\cot\eta_{res} = \kappa_{vb} + (k^2-\kappa_v^2)R^* \qquad \Gamma/2 = \kappa_{vb}/R^*$

$$a_{res}(k) = \frac{1}{-\kappa_{vb} + (k^2-\kappa_{vb}^2)R^*} \xrightarrow[k\to 0]{} \overbrace{\frac{1}{-\kappa_{vb}-\kappa_{vb}^2 R^*}}^{a_{res}} \xrightarrow[\kappa_{vs}\to 0]{} -\frac{1}{\kappa_{vb}}$$

bound states near threshold



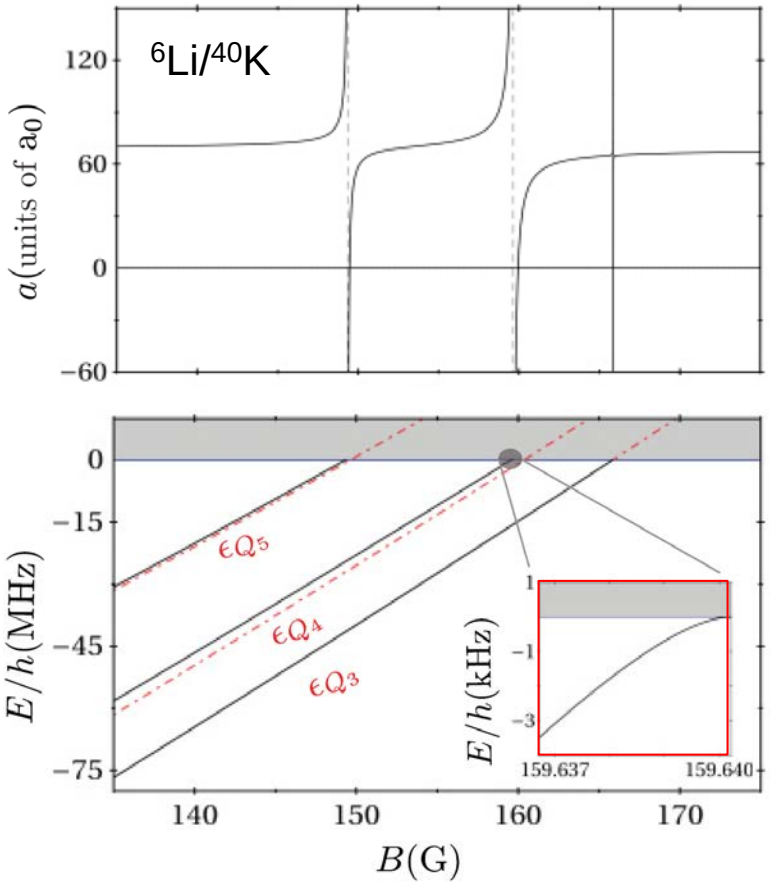


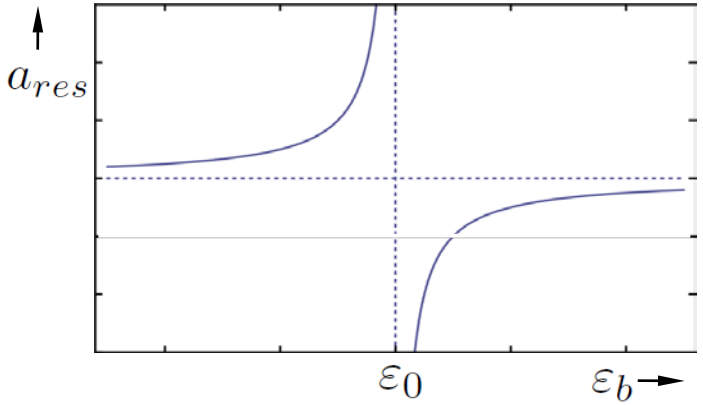
weakly bound level: $$a_{res} = \frac{1}{\kappa + \kappa^2 R^*} = -\frac{1/R^*}{(\varepsilon_b - \varepsilon_0)} > 0$$

threshold energy ($\kappa \to 0$)

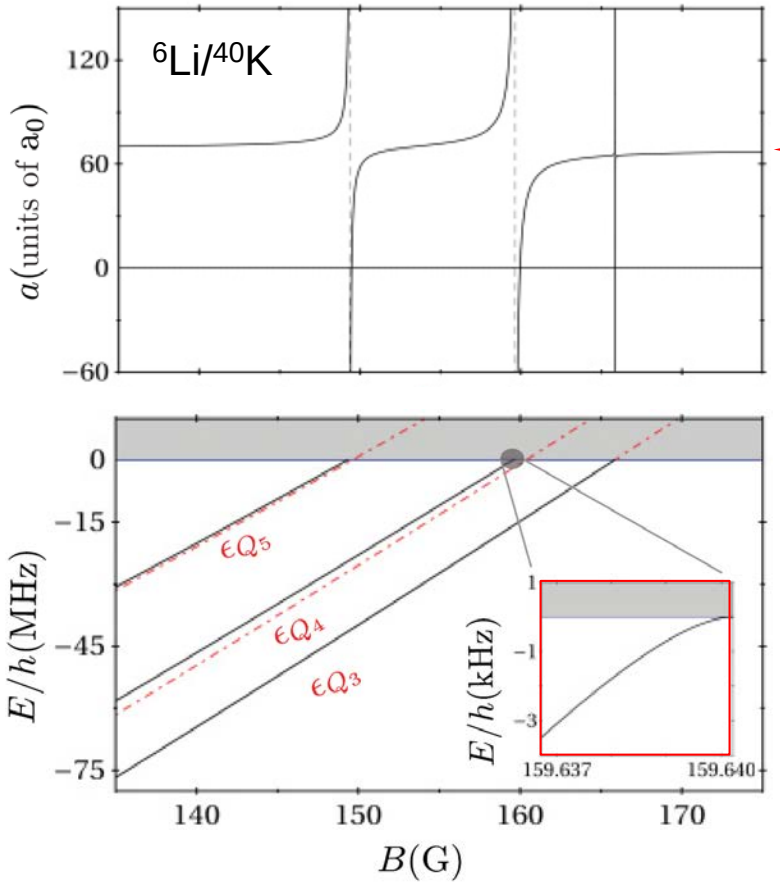
$$\Delta\varepsilon_v \equiv \varepsilon_v(B) - \varepsilon_v(B_t) \begin{cases} \Delta\varepsilon_v = -\kappa^2 < 0 \\ \Delta\varepsilon_v = \kappa_{vb}^2 > 0 \end{cases}$$

Relation to scattering length





Relation to scattering length



$$a_{res} = -\frac{1/R^*}{(\varepsilon_b - \varepsilon_0)} = -\frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$a(B) = a_{bg} - \frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$= a_{bg}\left[1 - \frac{(\hbar^2/m_r a_{bg} R^*)}{\mu_{\rm eff}(B - B_0)}\right]$$

$$a(B) = a_{bg}\left[1 - \frac{\Delta}{B - B_0}\right]$$

$$\Delta = \hbar^2/(m_r a_{bg} \mu_{\rm eff} R^*)$$

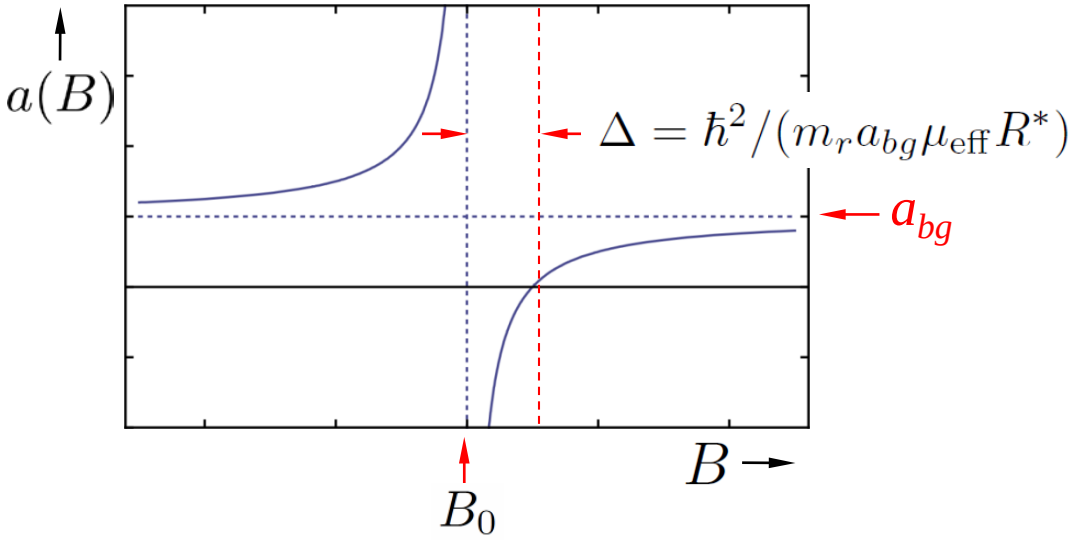
Relation to scattering length

$$a_{res} = -\frac{1/R^*}{(\varepsilon_b - \varepsilon_0)} = -\frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$a(B) = a_{bg}\left[1 - \frac{\Delta}{B - B_0}\right]$$

$$a(B) = a_{bg} - \frac{(\hbar^2/m_r R^*)}{\mu_{\rm eff}(B - B_0)}$$

$$= a_{bg}\left[1 - \frac{(\hbar^2/m_r a_{bg} R^*)}{\mu_{\rm eff}(B - B_0)}\right]$$

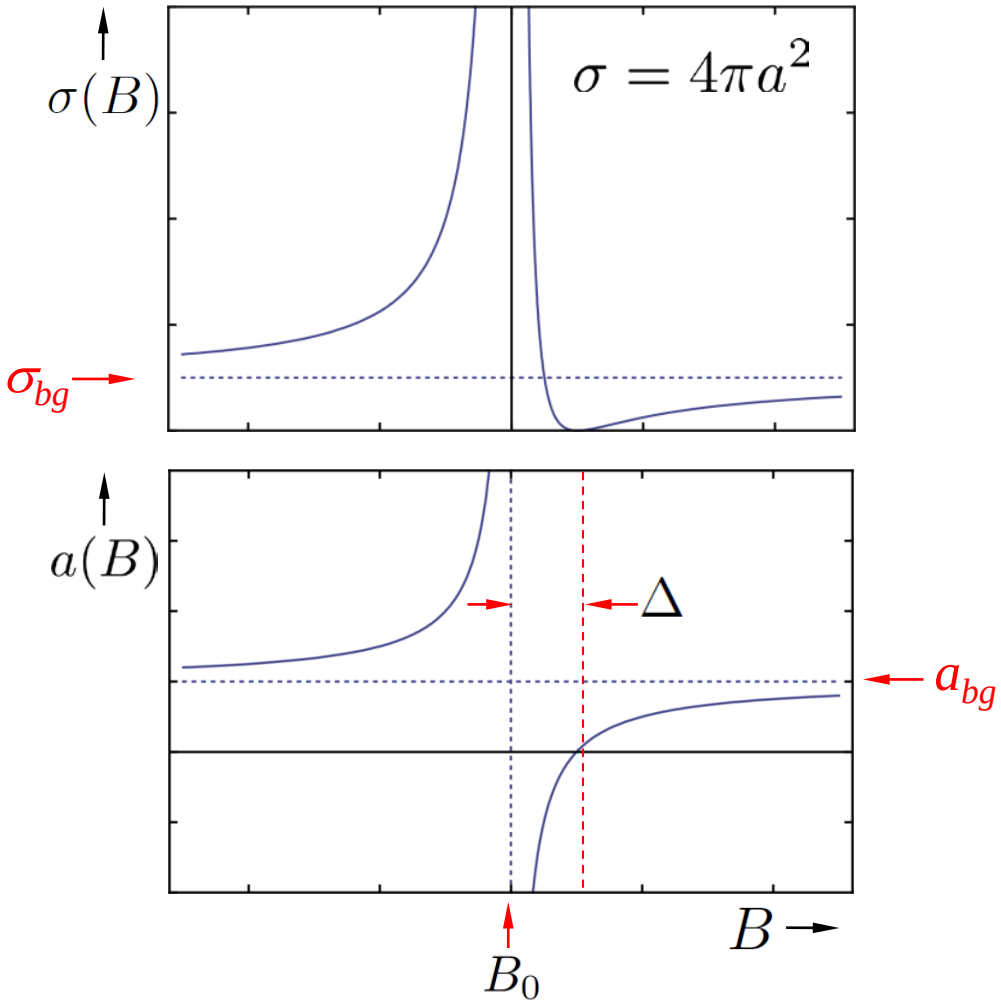


strength parameter:

$$s \equiv \frac{r_0}{R^*} = \frac{a_{bg}}{r_0}\frac{\mu_{\rm eff}}{E_0}\Delta$$

$$E_0 = \frac{\hbar^2}{m r_0^2}$$

Fano lineshape



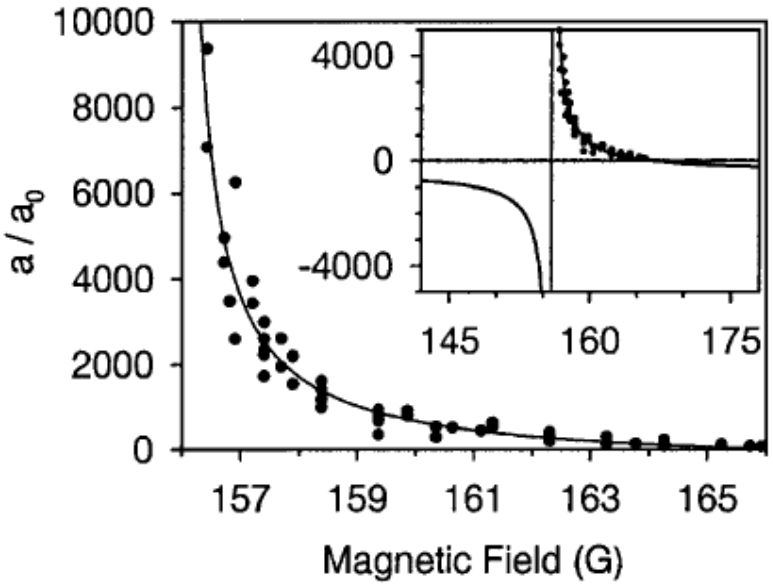
Narrow versus broad resonances

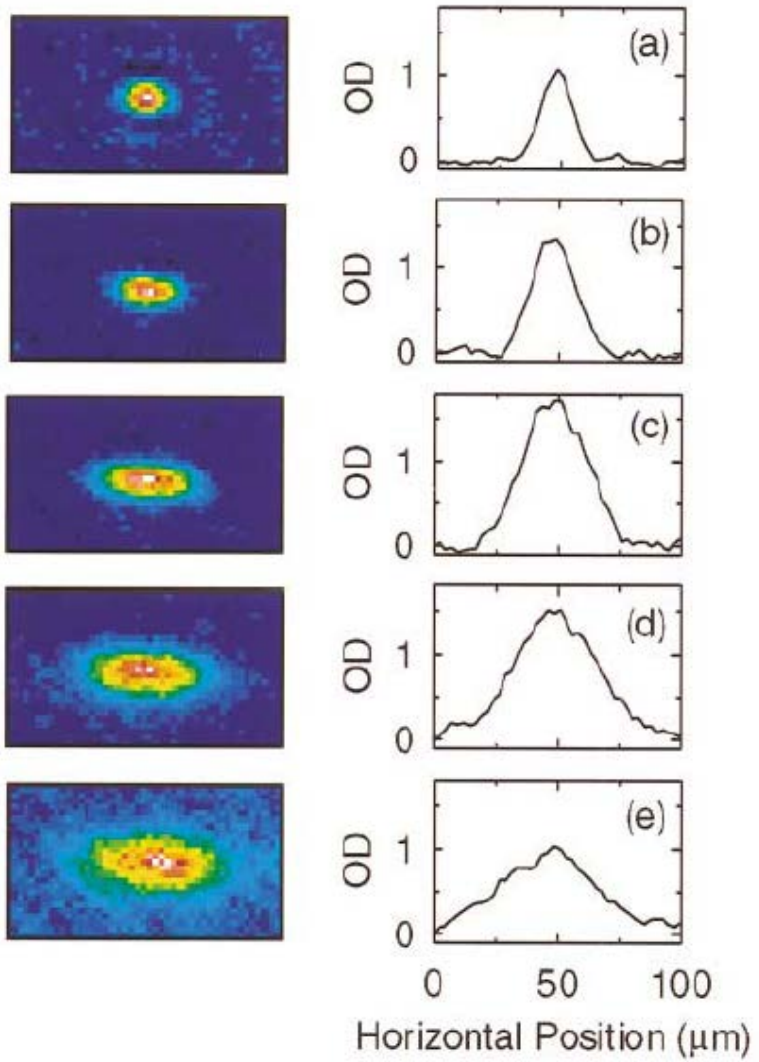
Importantly:

The meaning of broad and narrow depends on the context

- Broad as measured in magnetic field
- Broad as compared to the collisional band of energies
- Broad due to strong singlet-triplet coupling

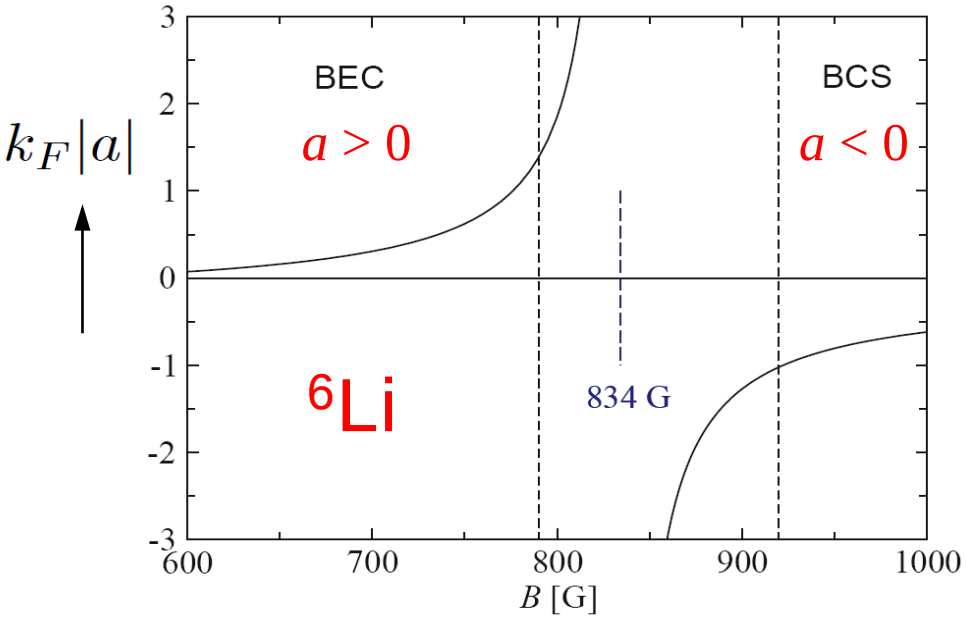
Rb-85





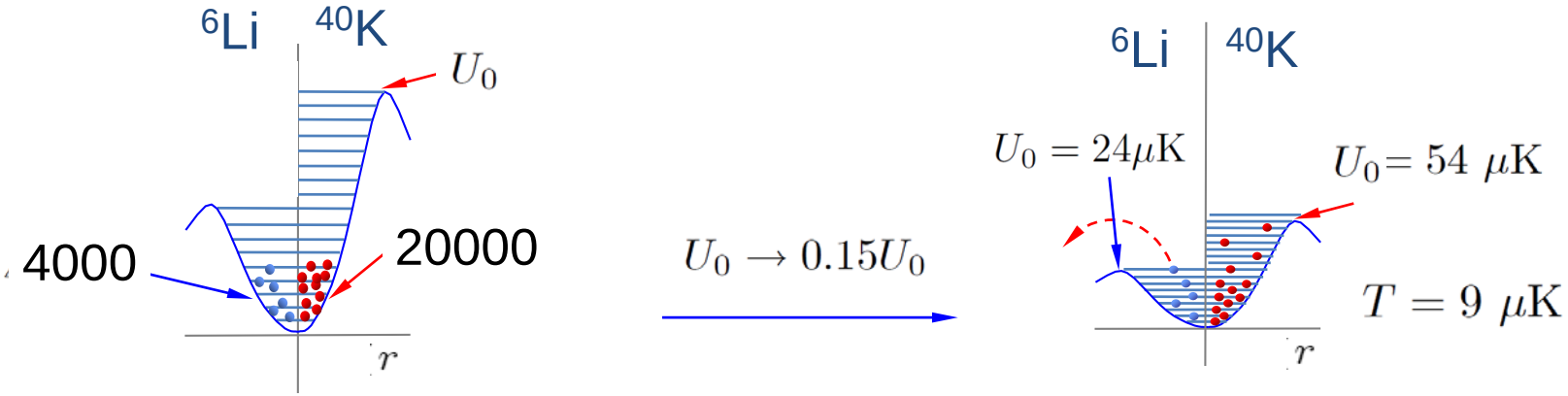
S.L. Cornish et al., PRL 85 (2000) 1995

Li-6

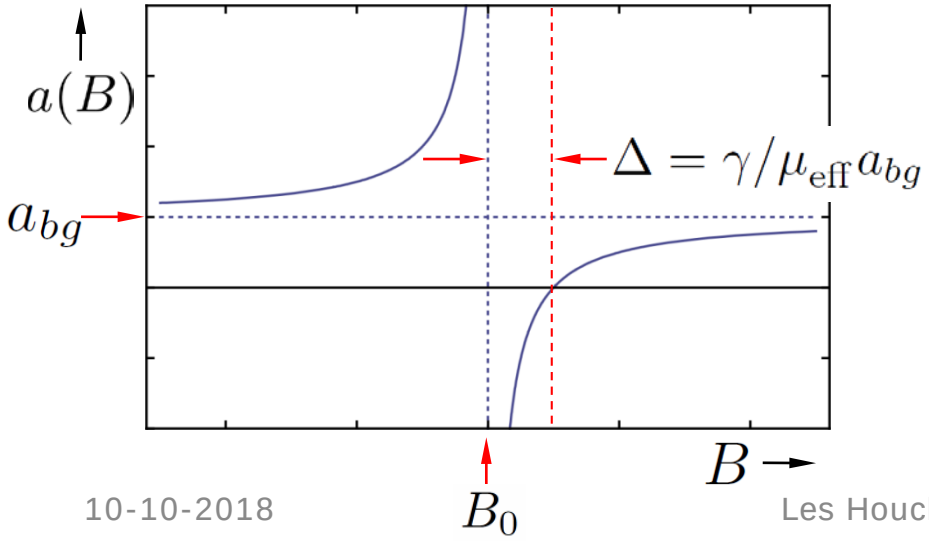


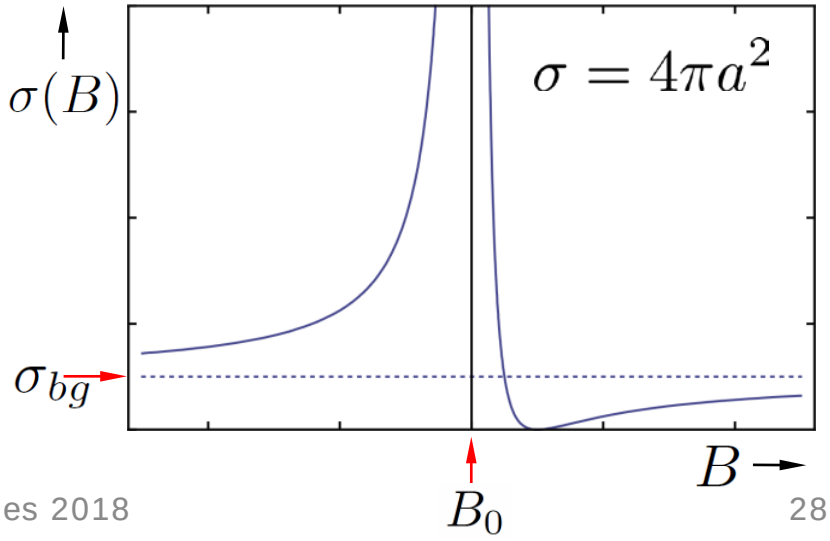
experimental

adiabatic expansion of Gaussian trap induces evaporation:

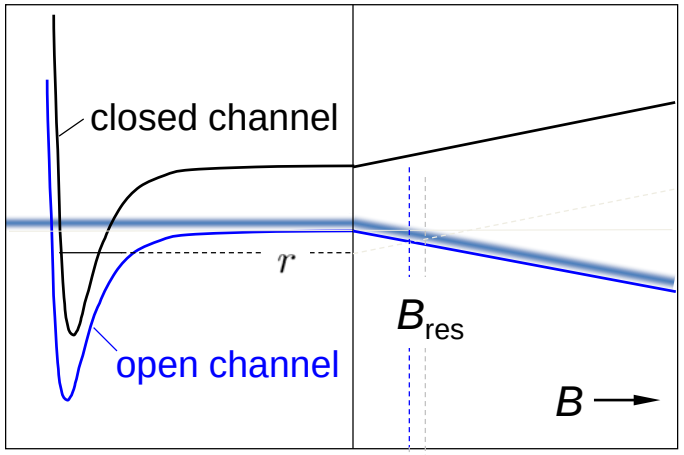


Measure evaporation of ^{6}Li due to collisions with ^{40}K





Feshbach scattering length

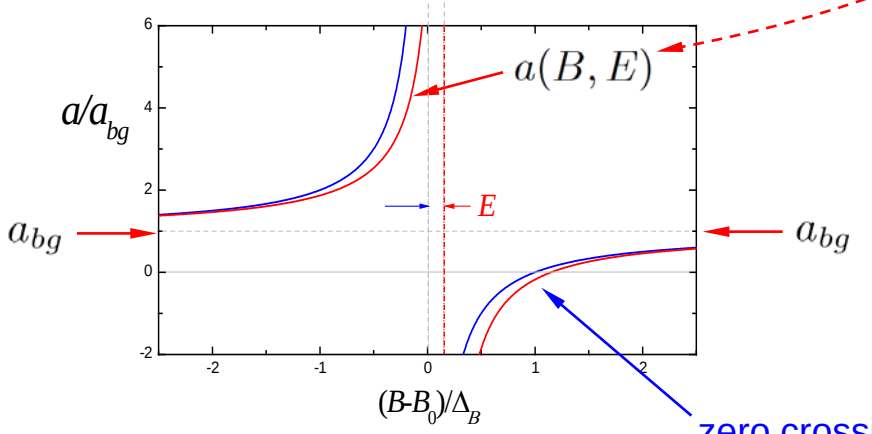


$$k \cot \eta_0 = -\frac{1}{a_{bg}(B,k) - a_{res}(B,k)} + \cdots$$

open channel (background)

closed channel (resonance)

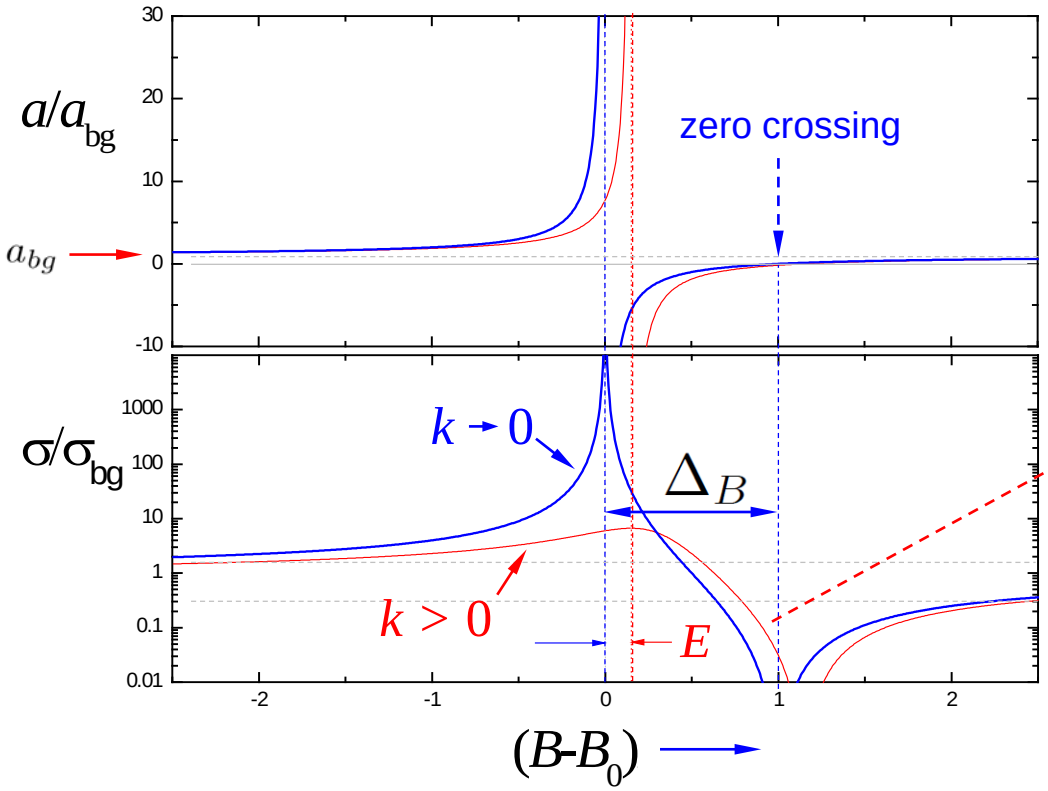
$$a(B,E) = a_{bg} + \frac{a_{bg}\mu_{\rm rel}\Delta_B}{E - \mu_{\rm rel}(B - B_0)}$$

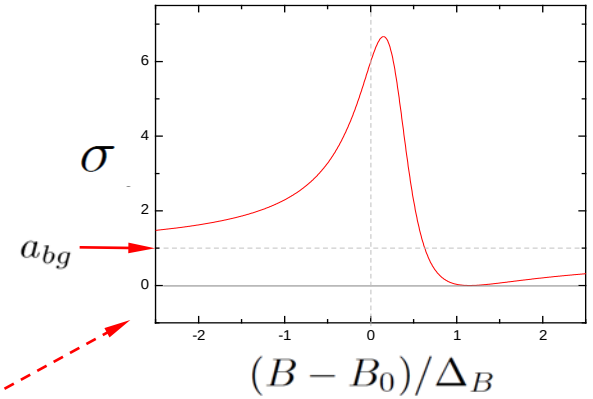


resonance strength parameter:

$$s_{res} = \frac{a_{bg}\mu_{\rm rel}\Delta_B}{r_0\left(\hbar^2/2\mu r_0^2\right)}$$

width from: *elastic cross section*



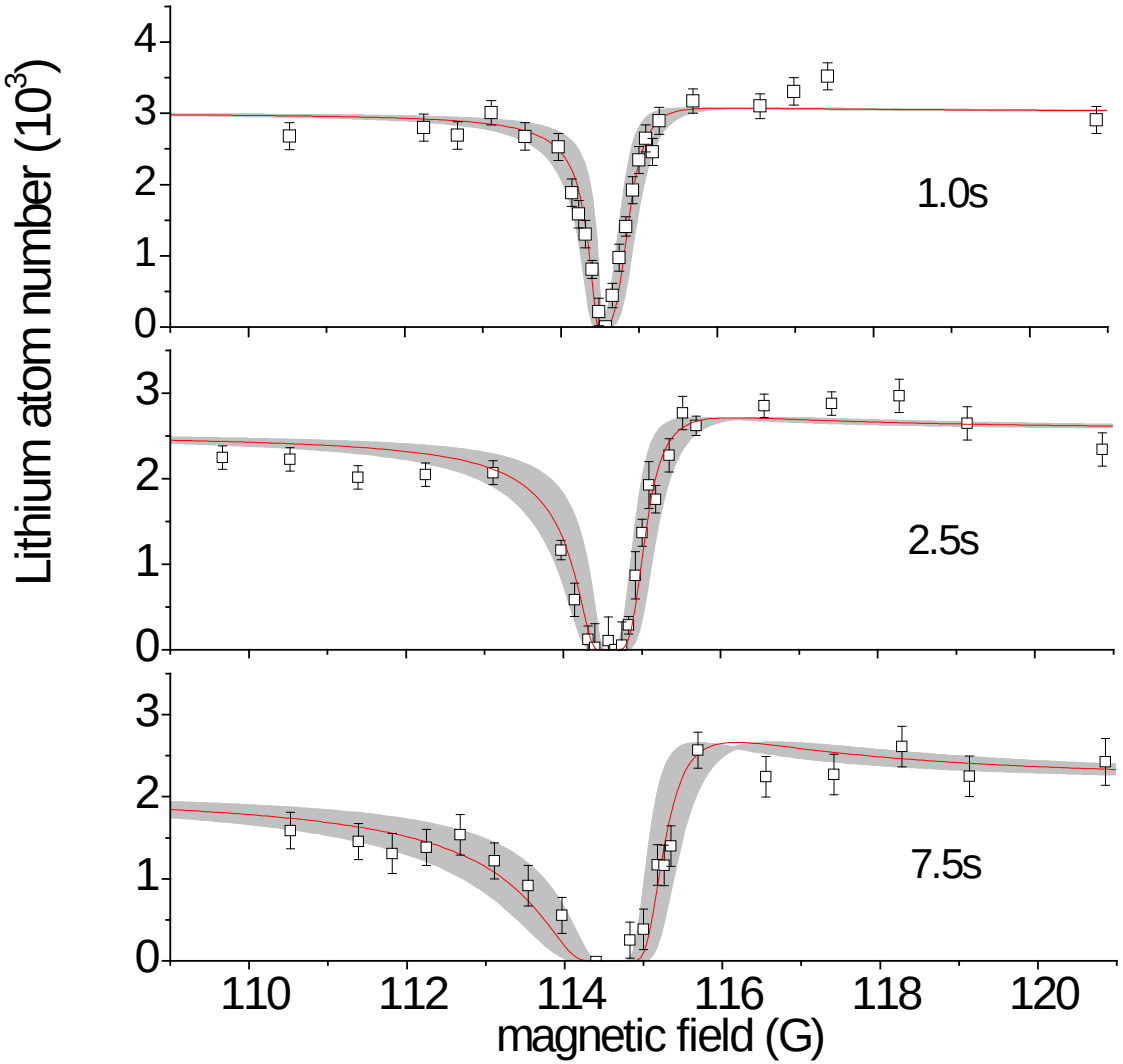


$$\sigma = 4\pi \frac{[a_{bg} - a_{res}]^2}{1 + k^2 [a_{bg} - a_{res}]^2}$$

on resonance ($a \to \infty$ *unitarity limited scattering* ($\sigma = \frac{4\pi}{k^2}$)

for $a_{bg} \neq 0$ asymmetric line shape: *Fano profile*

characterization of a Feshbach resonance



position: $B_0 = 114.47(5)$ G

width: $\Delta B = 1.5(5)$ G

T.G. Tiecke, et al.
PRL 104, 053202 (2010)

End of Les Houches Lectures

Thank you