

The bizarre one-dimensional Quantum World

T. Giamarchi

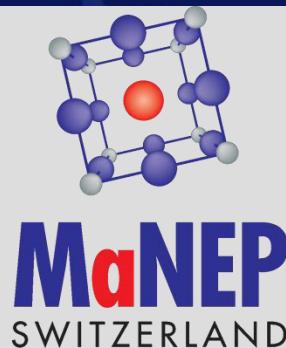
<http://dqmp.unige.ch/giamarchi/>



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DE GENÈVE

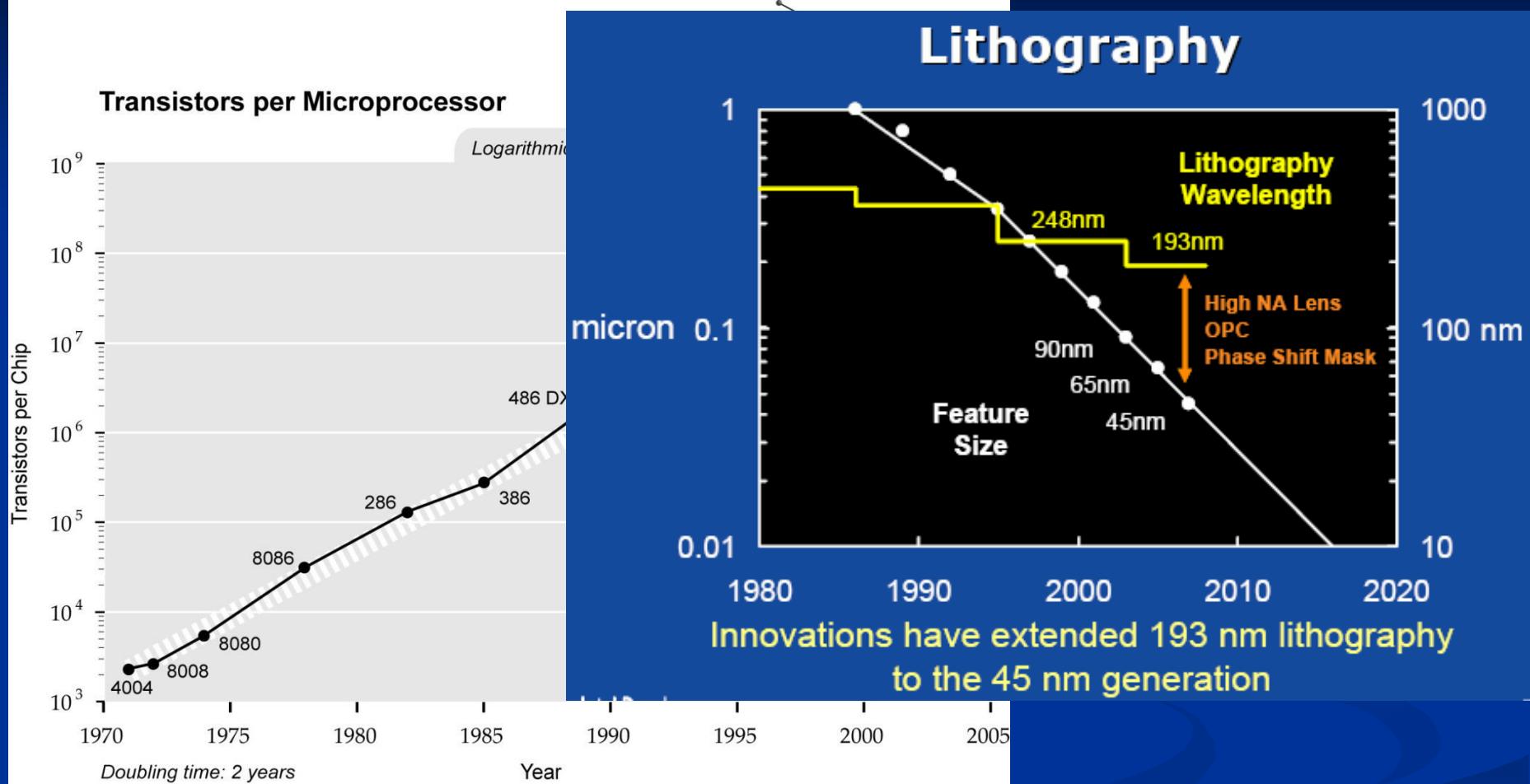
FNSNF

FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION



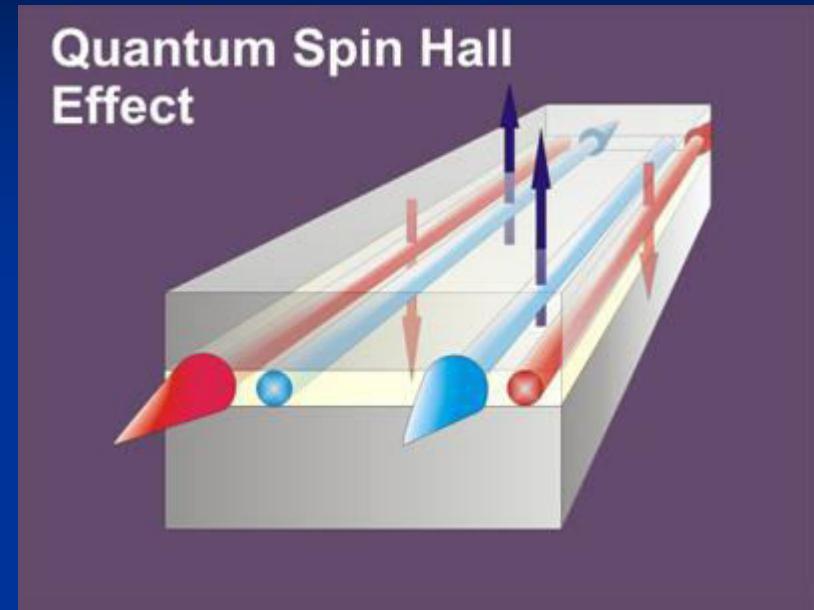
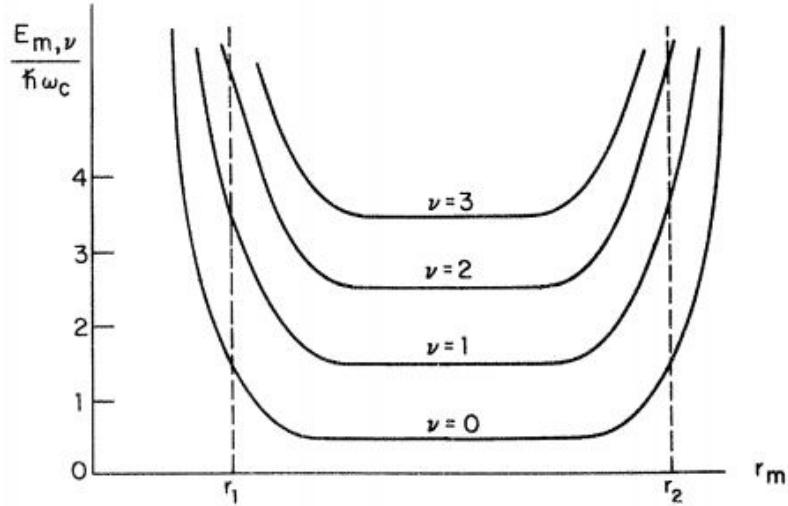
Why look at reduced dimensionality

Future electronic



Need to worry about reduced dimensionality

Physics at the edge



Presence of edge
(B. I. Halperin)



Quantum hall effect
Topological insulators....

One dimensional quantum systems

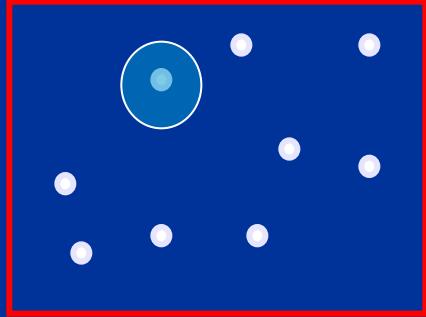
A good reason to work on 1D

However, my personal reason for working on one-dimensional problems is merely that they are **fun**. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

Freeman Dyson (1967)

One dimension is specially interesting

- No individual excitation can exist (only collective ones)



- Strong quantum fluctuations

$$\psi = |\psi| e^{i\theta}$$

Difficult to order

Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

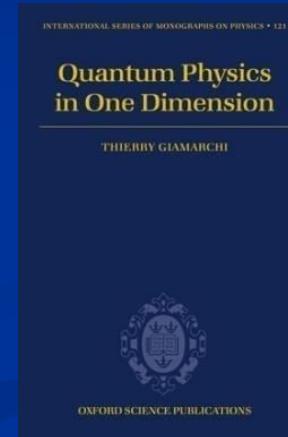
References

TG, arXiv/0605472 (Salerno lectures)

TG, Quantum physics in one dimension, Oxford (2004)

M. Cazalilla et al.,
Rev. Mod. Phys. 83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)



High dimension: Fermi liquids



References on fermi liquids

- Basic course on many-body physics
[http://dqmp.unige.ch/giamarchi/local/people/
thierry.giamarchi/pdf/many-body.pdf](http://dqmp.unige.ch/giamarchi/local/people/thierry.giamarchi/pdf/many-body.pdf)
- Lectures Les houches (Singapore) 2009, TG
[arXiv:1007.1030](#)
- Lectures Les Houches 2010, A. Georges + TG
[arXiv:1308.2684](#)

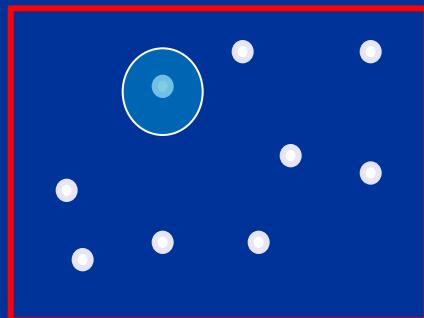
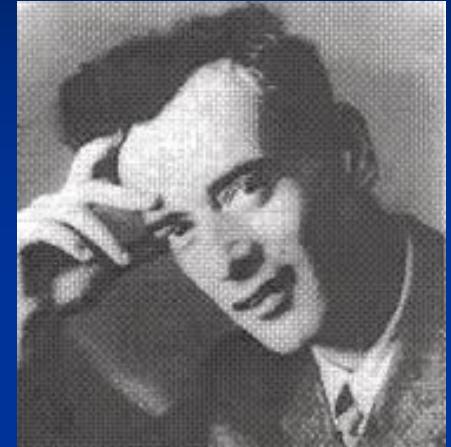
Fermi liquid theory

- Shown perturbatively in U
- Much more general and robust

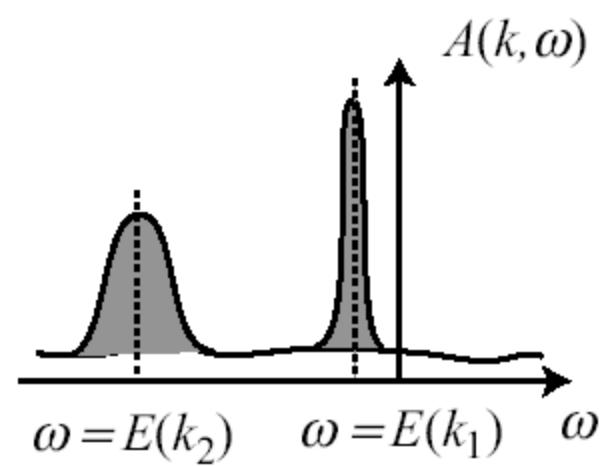
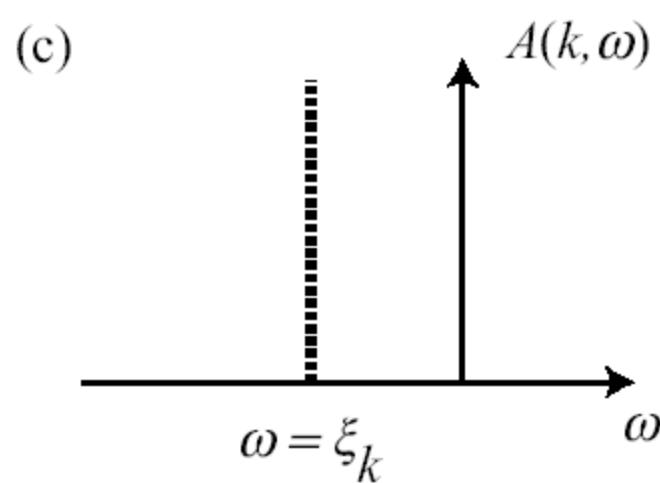
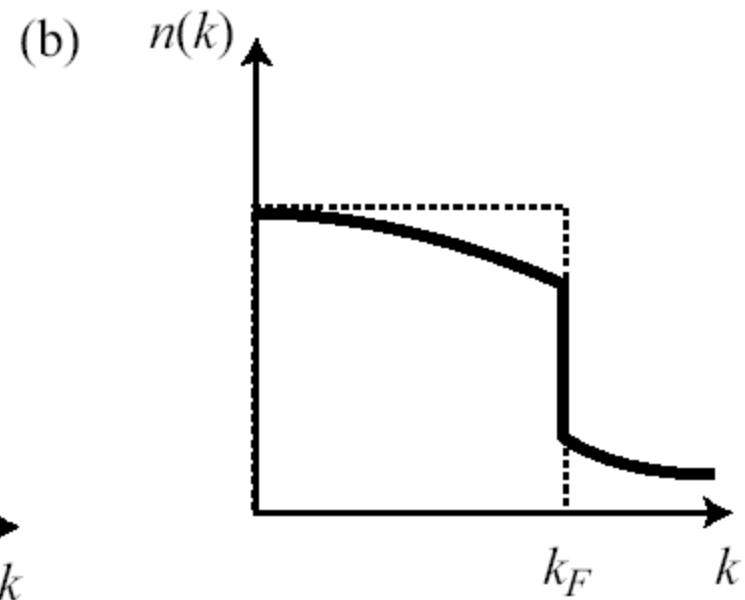
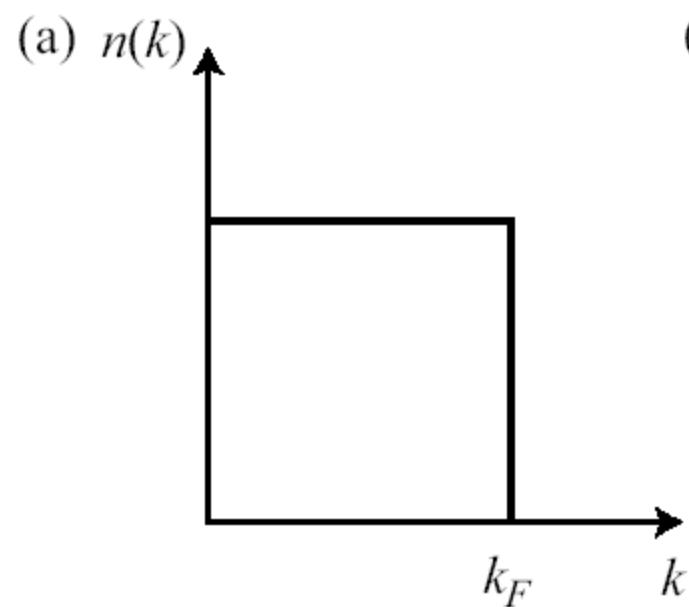
Element	m^*/m	χ/χ_0
Nb	2	1
^3He	6	20
Heavy fermion	100	100

Effect of interactions

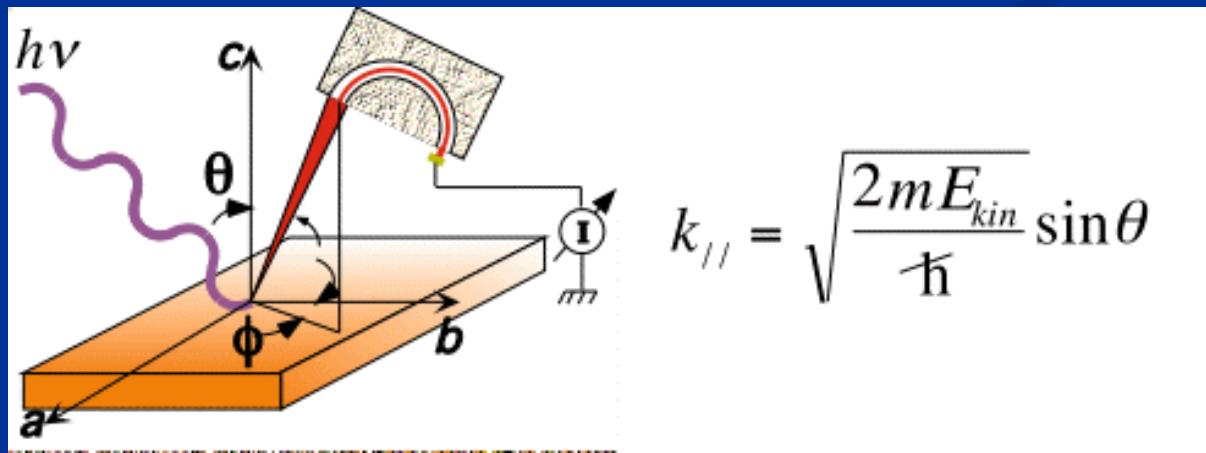
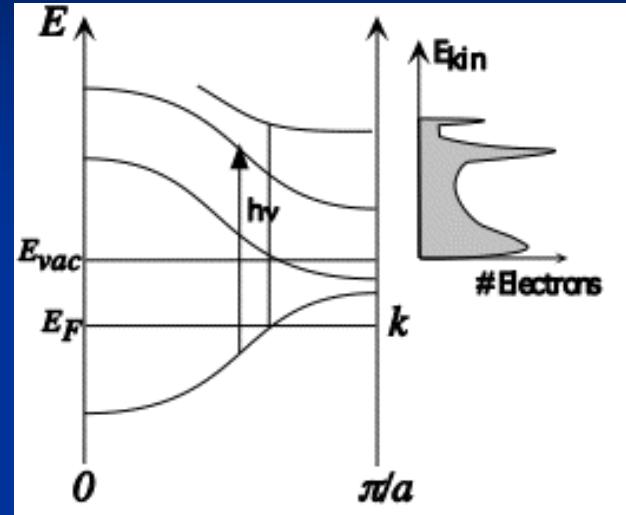
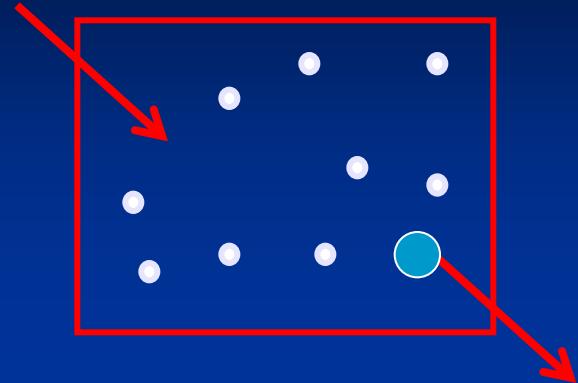
- Landau Fermi liquid
- Individual fermionic excitations exist (quasiparticles)



$$m \neq m^*$$

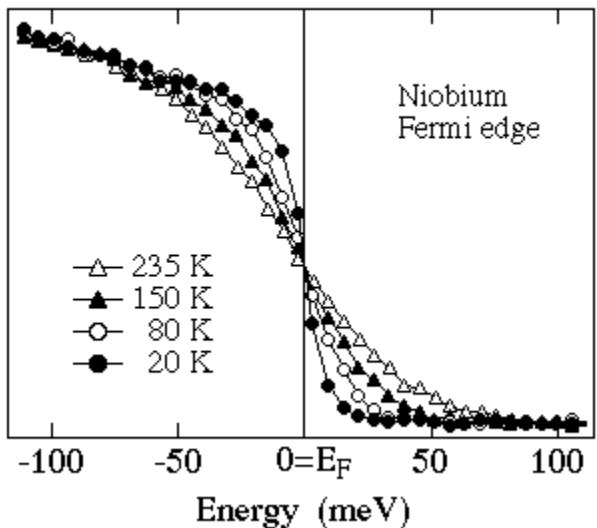


Photoemission

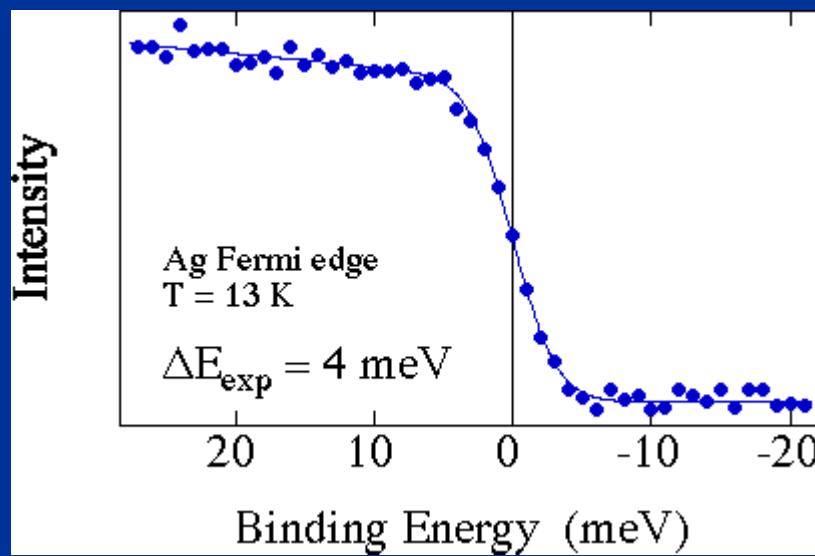


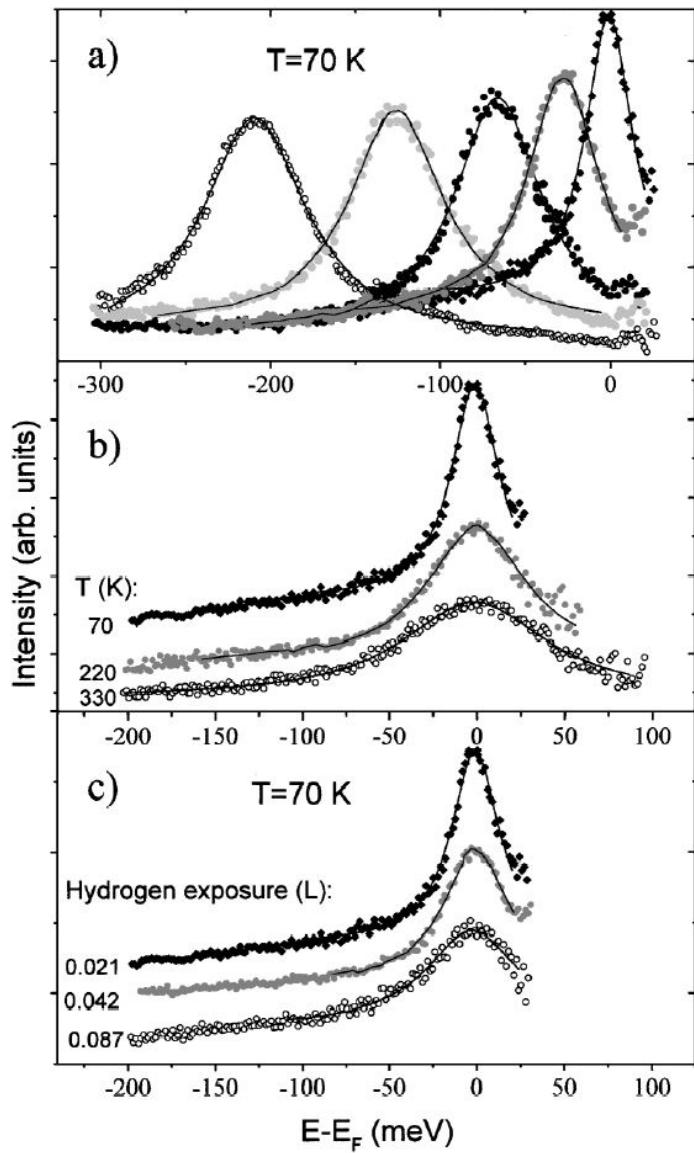
$$k_{\parallel} = \sqrt{\frac{2mE_{kin}}{\hbar}} \sin \theta$$

Measures
Spectral
function
 $A(k, \omega)$



(M. Grioni)



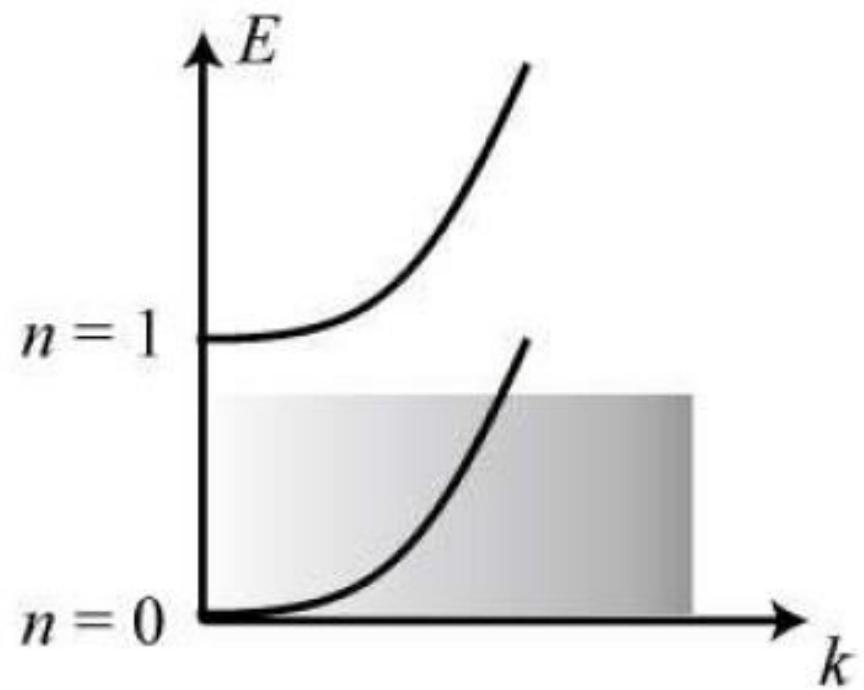
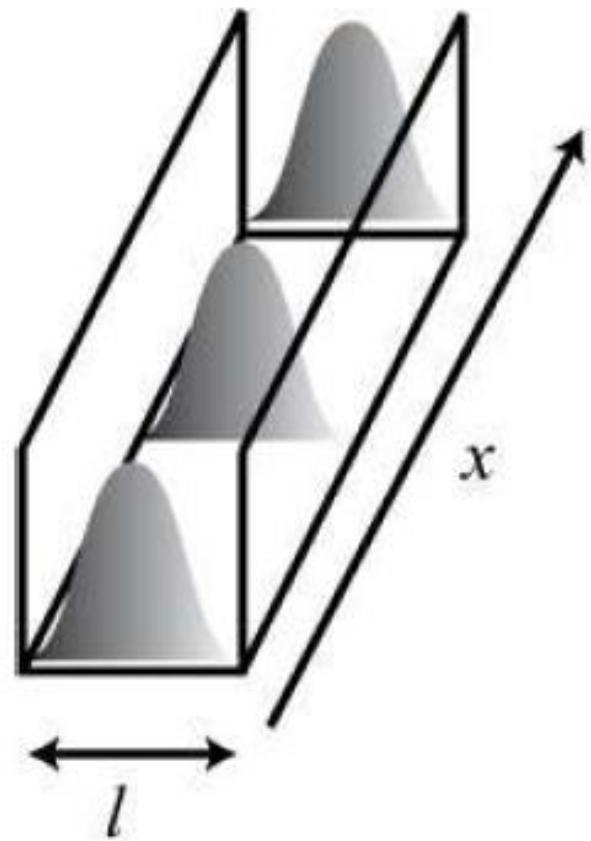


T. Valla et al. PRL 83
2085 (1999)

Mo(110) surface

FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cutoff. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the (a) binding energy, (b) temperature, and (c) hydrogen exposure is shown.

What does “1D” means in the real (3D ?) world



$$E = \frac{k_x^2}{2m} + \frac{k_y^2}{2m}$$

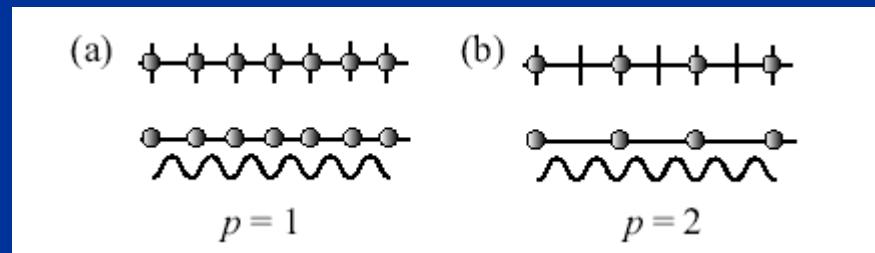
$$k_y = \frac{2\pi n}{L_y}$$

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger (\nabla\psi)}{2M} + \frac{1}{2} \int dx dx' V(x - x') \rho(x) \rho(x') - \mu \int dx \rho(x)$$

- Lattice:



$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

How to treat ?



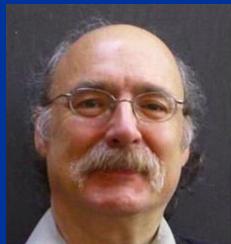
■ ``Standard'' many body theory



■ Exact Solutions (Bethe ansatz)



■ Field theories (bosonization, CFT)



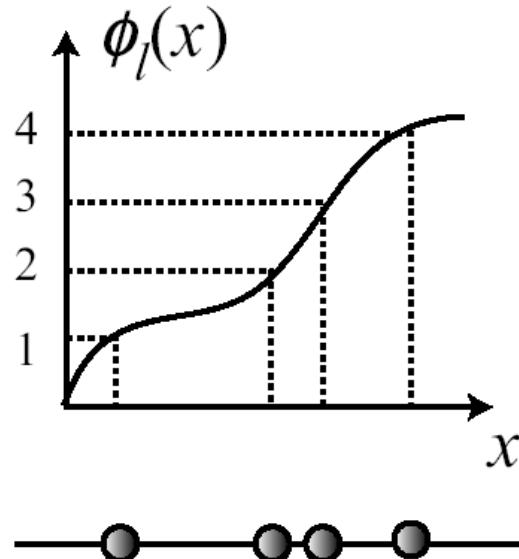
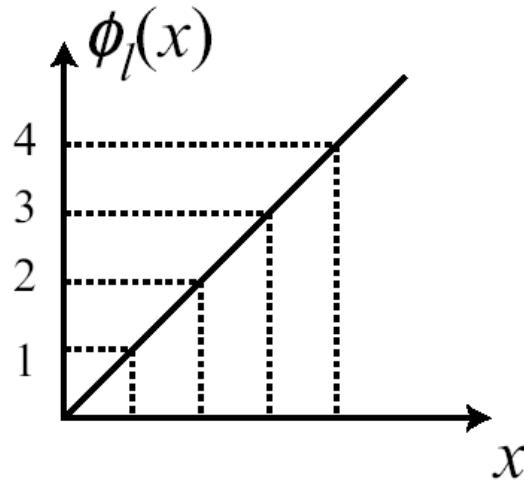
■ Numerics (DMRG, MC, etc.)



Labelling the particles

$$\begin{aligned}\rho(x) &= \sum_i \delta(x - x_i) \\ &= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)\end{aligned}$$

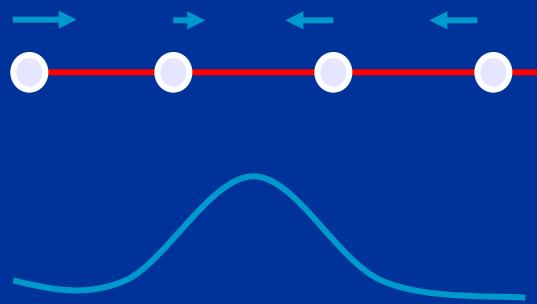
1D: unique way
of labelling



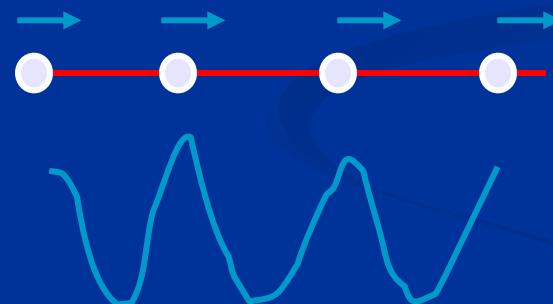
$$\phi_l(x) = 2\pi\rho_0x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi} \nabla \phi(x) \right] \sum_p e^{i2p(\pi\rho_0x - \phi(x))}$$

$\phi(x)$ varies slowly



$$q \sim 0$$



CDW

$$q \sim 2\pi\rho_0$$

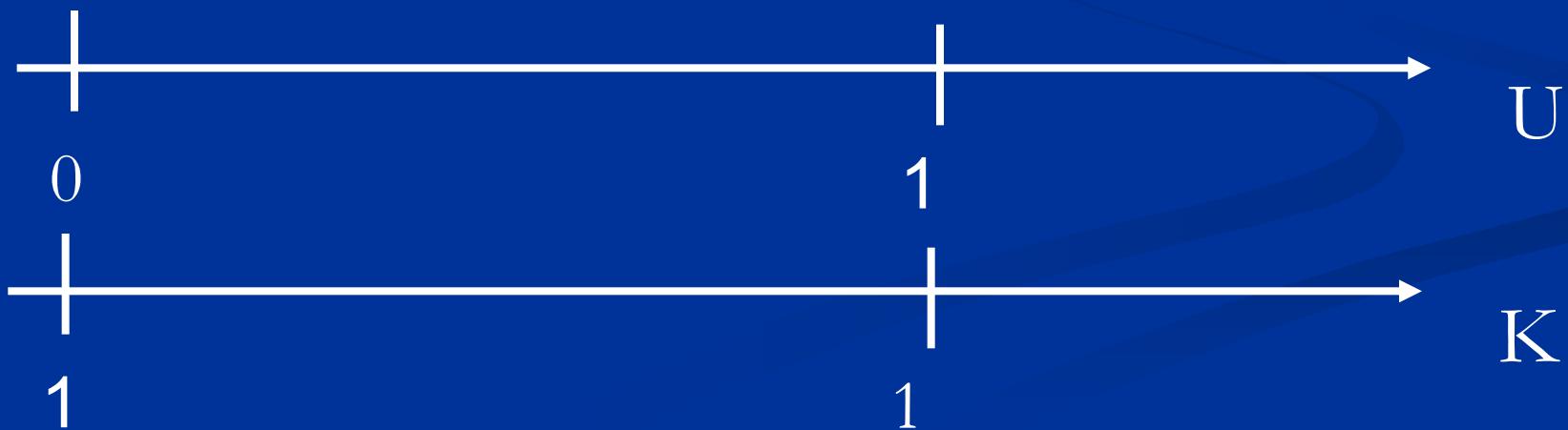
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ : superfluid phase

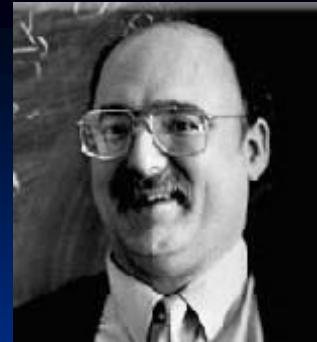
$$[\frac{1}{\pi} \nabla \phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum
fluctuations

$$H = \frac{\hbar}{2\pi} \int dx \left[\frac{uK}{\hbar^2} (\pi\Pi(x))^2 + \frac{u}{K} (\nabla\phi(x))^2 \right]$$



Luttinger liquid concept

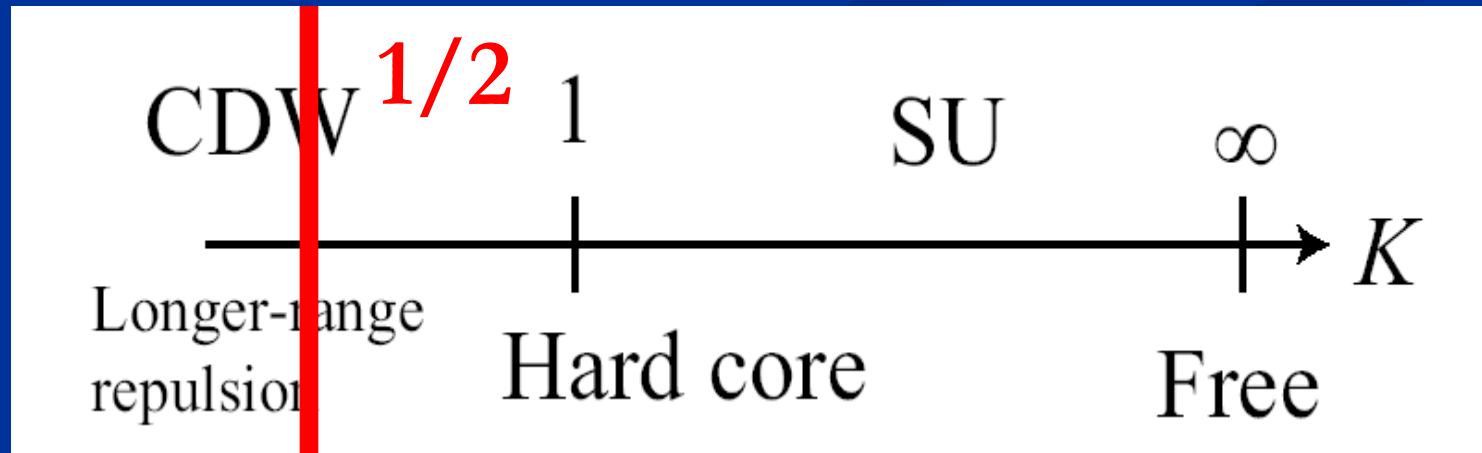


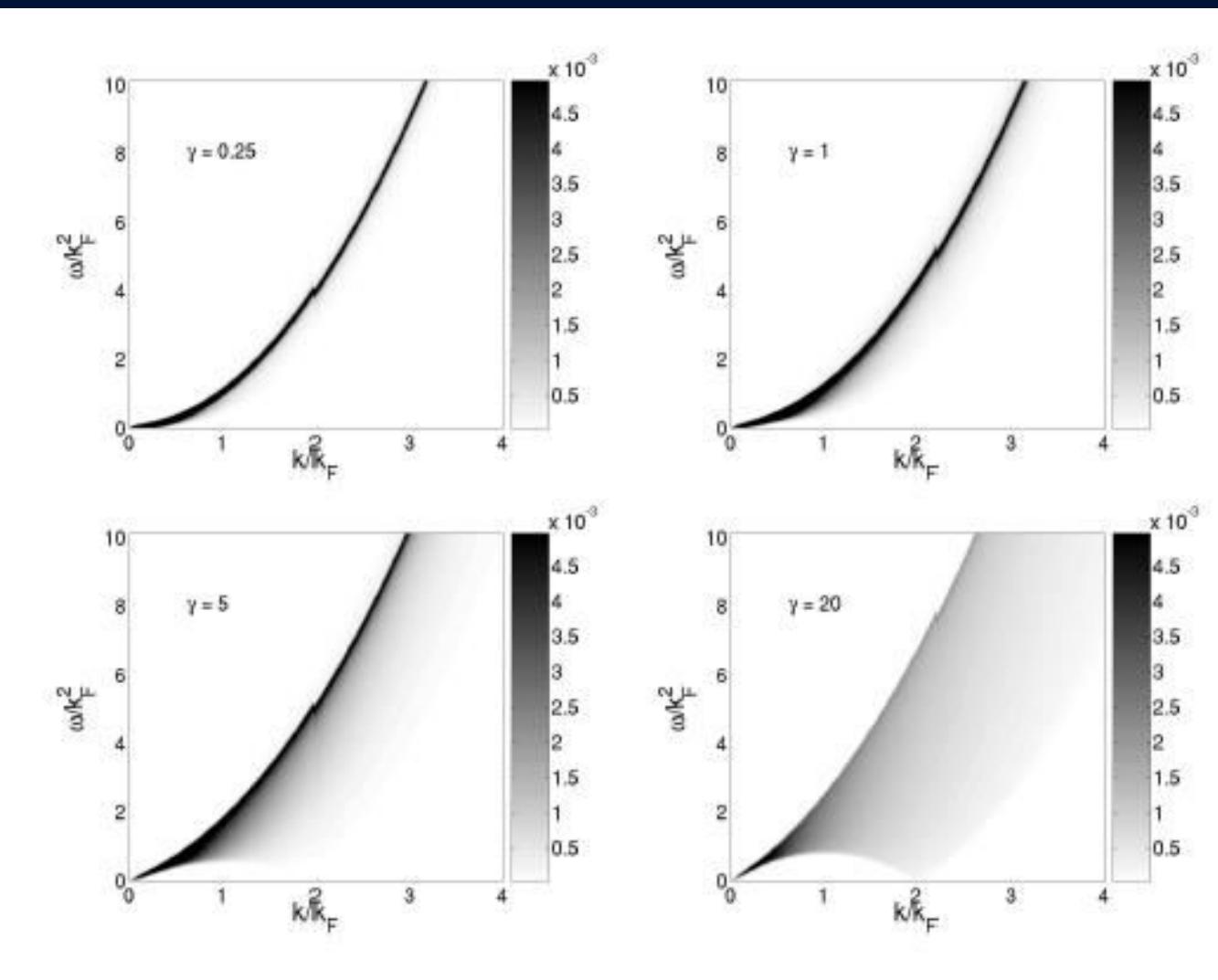
- How much is perturbative ?
- Nothing (Haldane):
provided the correct u, K are used
- Low energy properties: Luttinger liquid
(fermions, bosons, spins...)

Correlations

$$\langle \psi(r)\psi^\dagger(0) \rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \dots$$

$$\langle \rho(r)\rho(0) \rangle = \rho_0^2 + \frac{K}{2\pi^2} \frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x) \left(\frac{1}{r}\right)^{2K} + \dots$$





$S(q, !)$ J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory



Spins

Use boson or fermions mapping

$$S^+ = (-1)^i e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^z = \frac{-1}{\pi} \nabla \phi + (-1)^i \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x, 0) S^z(0, 0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x, 0) S^-(0, 0) \rangle = C_3 \left(\frac{1}{x}\right)^{2K + \frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

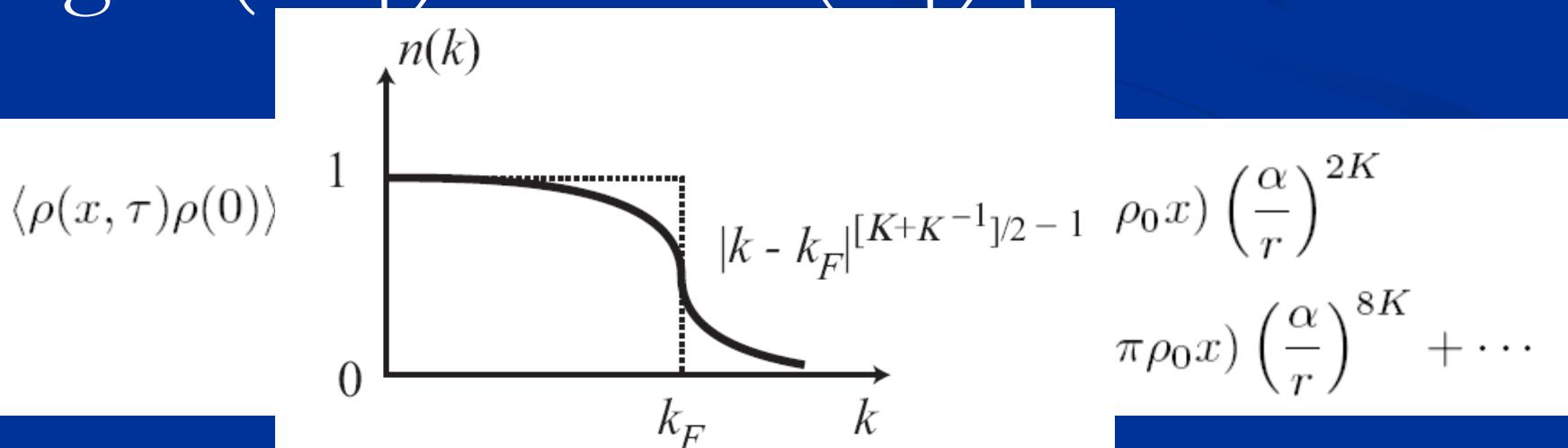
Non universal exponents K(h,J)

Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi} \nabla \phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles



Calculation of Luttinger parameters

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: $u K$
- Specific heat : T/u
- Etc.

Tonks limit



$U = 1$: spinless fermions

Not for $n(k)$: $\Psi_F \neq \Psi_B$

Free fermions: $\langle \rho(x)\rho(0) \rangle \propto \cos(2k_F x) \left(\frac{1}{x}\right)^2$

$K=1$

Note: $\langle \psi_B(x)\psi_B(0)^\dagger \rangle \propto \left(\frac{1}{x}\right)^{1/2}$

Remarkable **universal** physics: Tomonaga-Luttinger Liquid

- Good excitations: collective (sound waves) velocity **u**
- Powerlaw correlation functions depending on a single parameter **K**

$$\langle \psi(x)\psi^\dagger(0) \rangle = A_1 \left(\frac{a}{x} \right)^{1/(2K)}$$

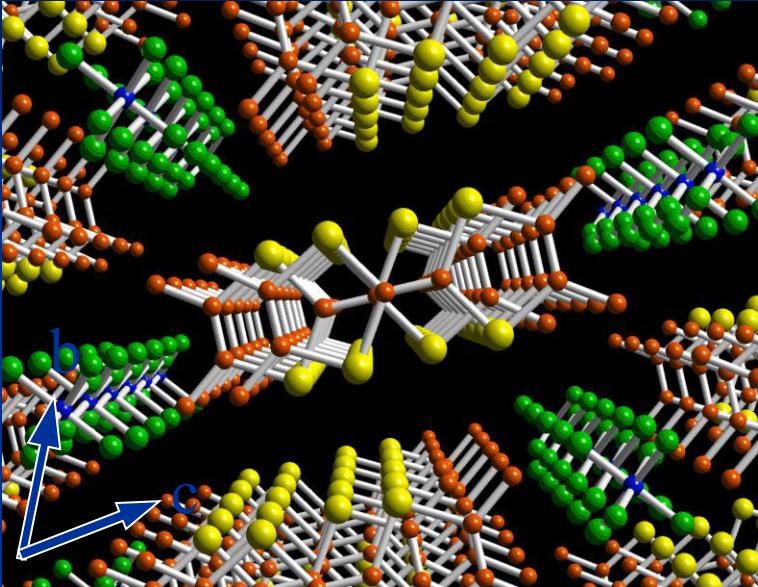
$$\langle \rho(x)\rho(0) \rangle = \rho_0^2 - \frac{K}{2\pi^2 x^2} + A_2 \cos(2\pi\rho_0 x) \left(\frac{a}{x} \right)^{2K}$$

Test of Tomonaga-Luttinger liquids

TG, Int J. Mod. Phys. B 26 1244004 (2012)

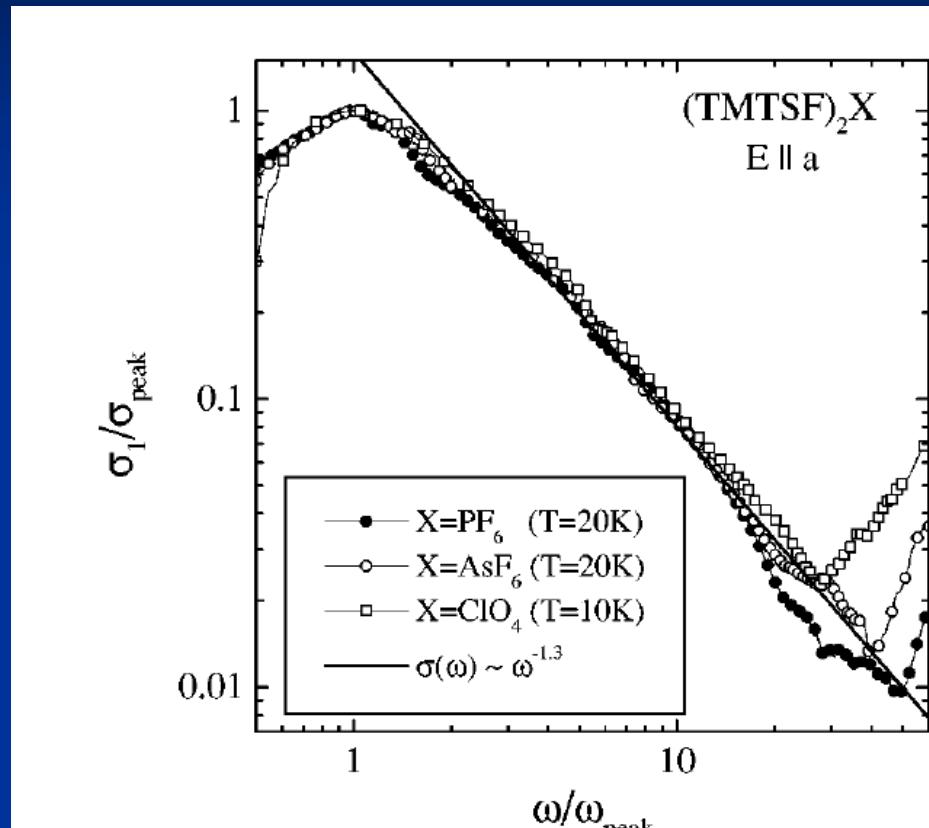
TG, C. Rend. Acad. Sci 17 322 (2017)

Organic conductors



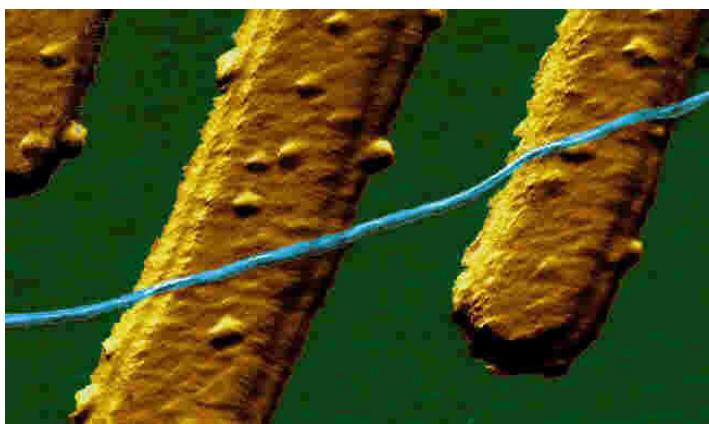
$$\sigma(\omega) \sim \omega^{-\nu}$$

TG PRB (91) :
Physica B 230 (1996)

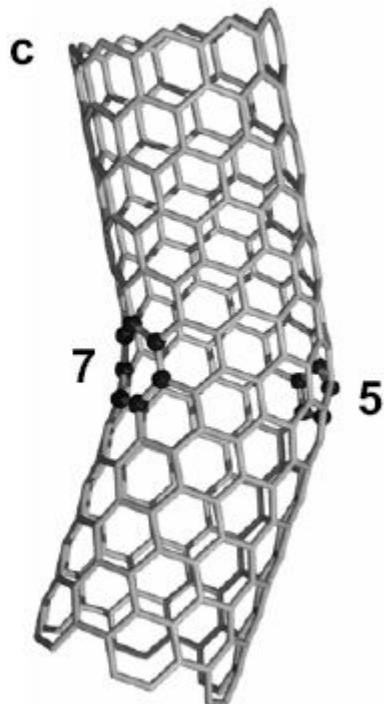
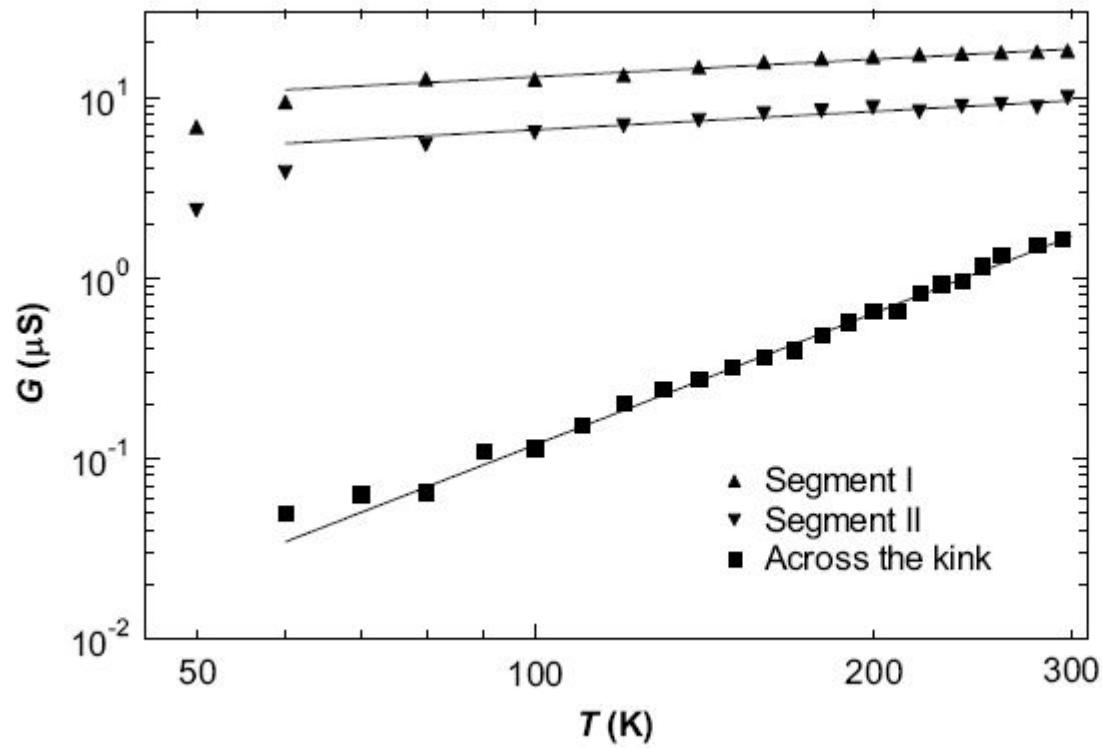


A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!

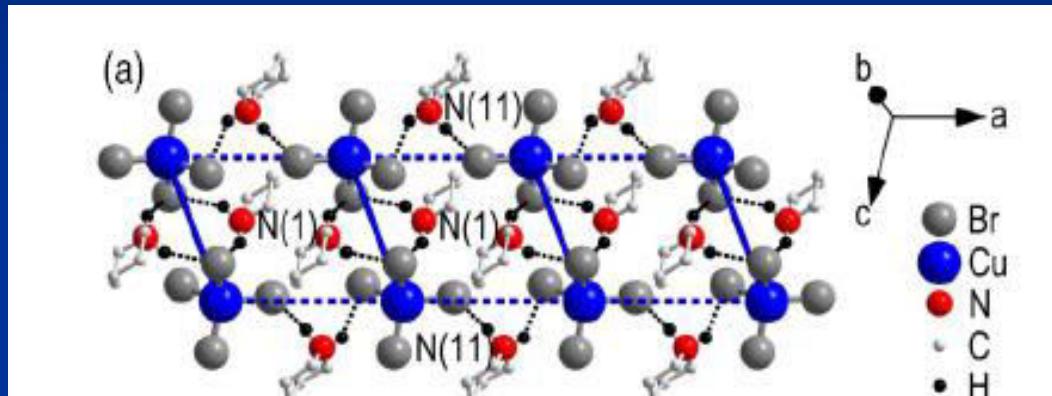


Z. Yao et al. Nature 402
273 (1999)



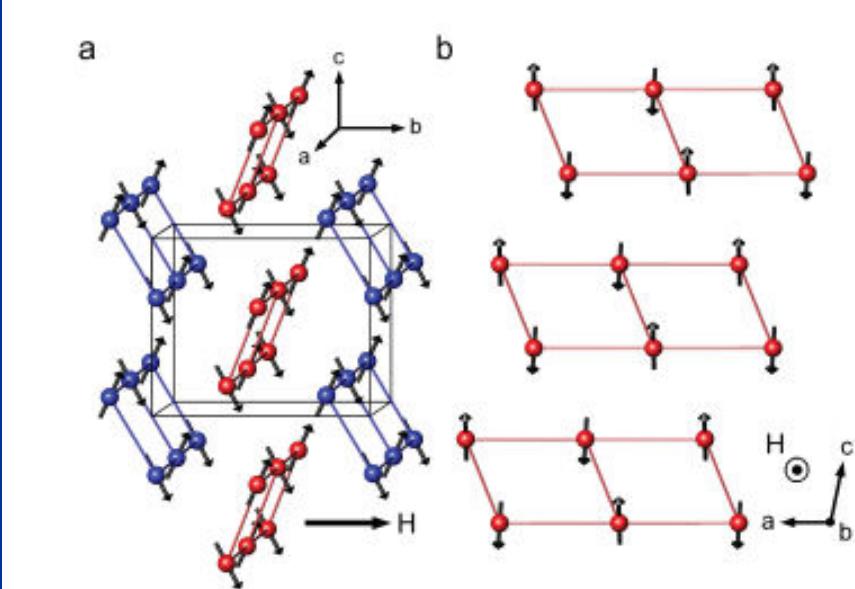
Spin chains and ladders

B. C. Watson et al., PRL 86 5168 (2001)

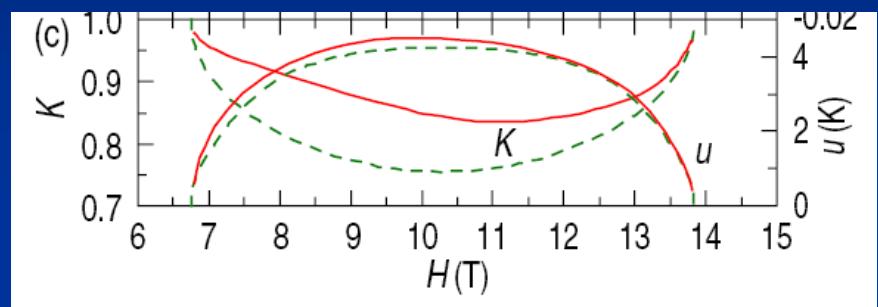
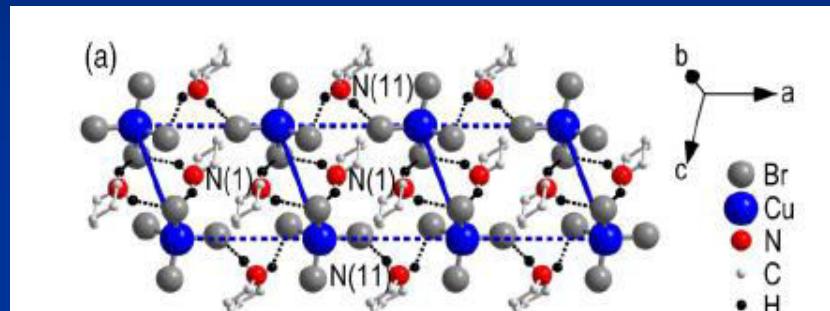


M. Klanjsek et al.,
PRL 101 137207 (2008)

B. Thielemann et al.,
PRB 79, 020408® 2009



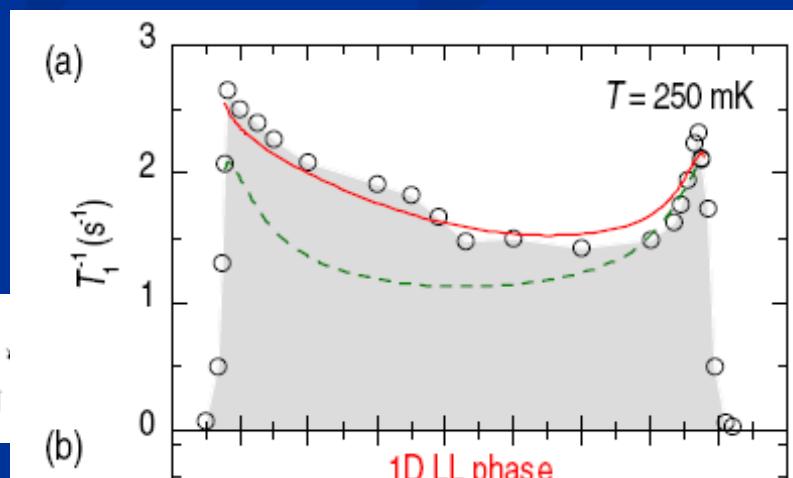
Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

■ NMR relaxation rate:

$$L_{I-I}^I = \frac{\kappa^B w}{4\lambda_5 V_5^T V_i^0} \cos\left(\frac{4K}{w}\right) B\left(\frac{4K}{I}, I - \frac{5K}{I}\right) \left(\frac{w}{5wL}\right)_{(I \setminus 5K)-I}$$



Cold atoms



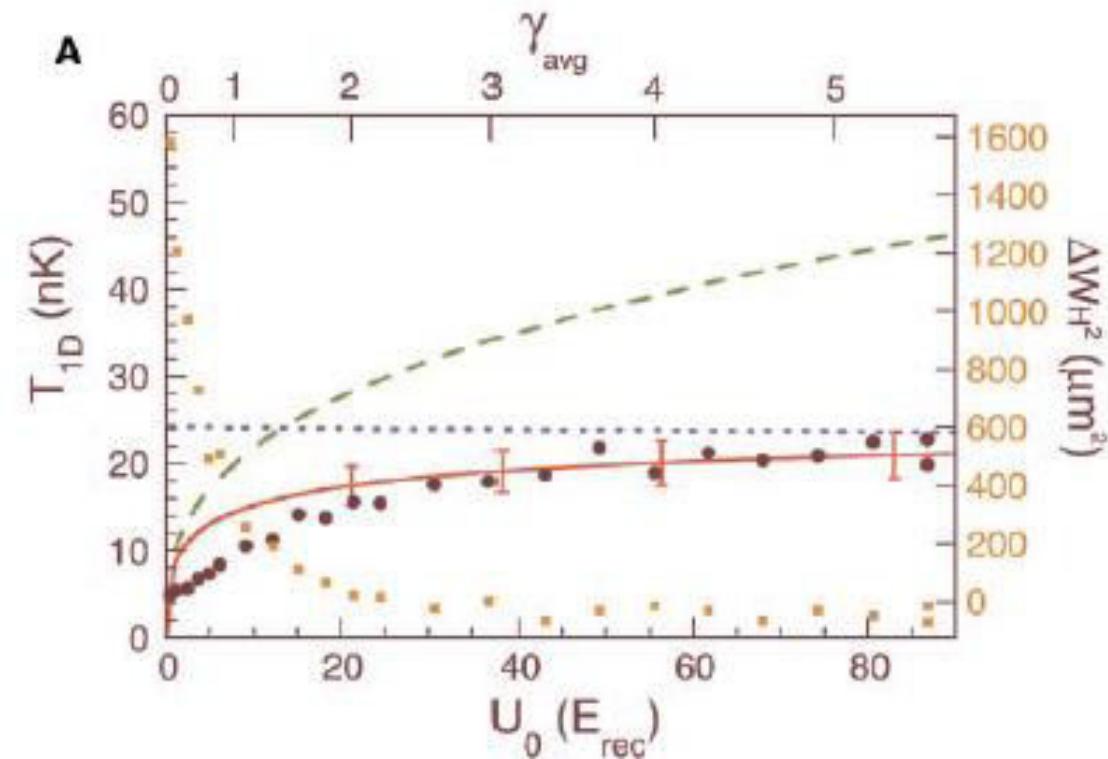
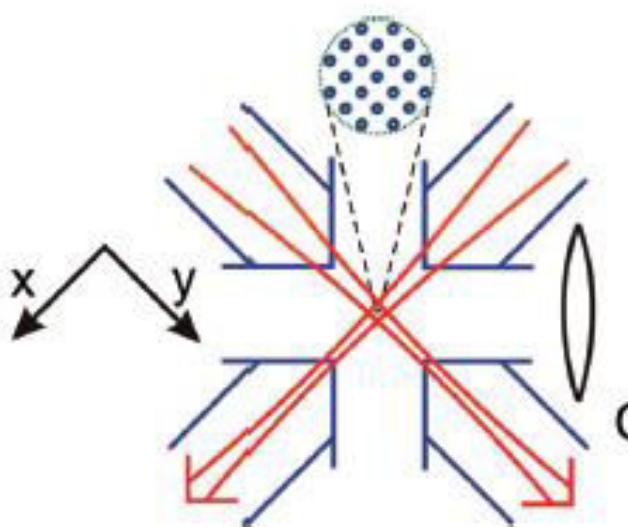
Bosons (continuum)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004

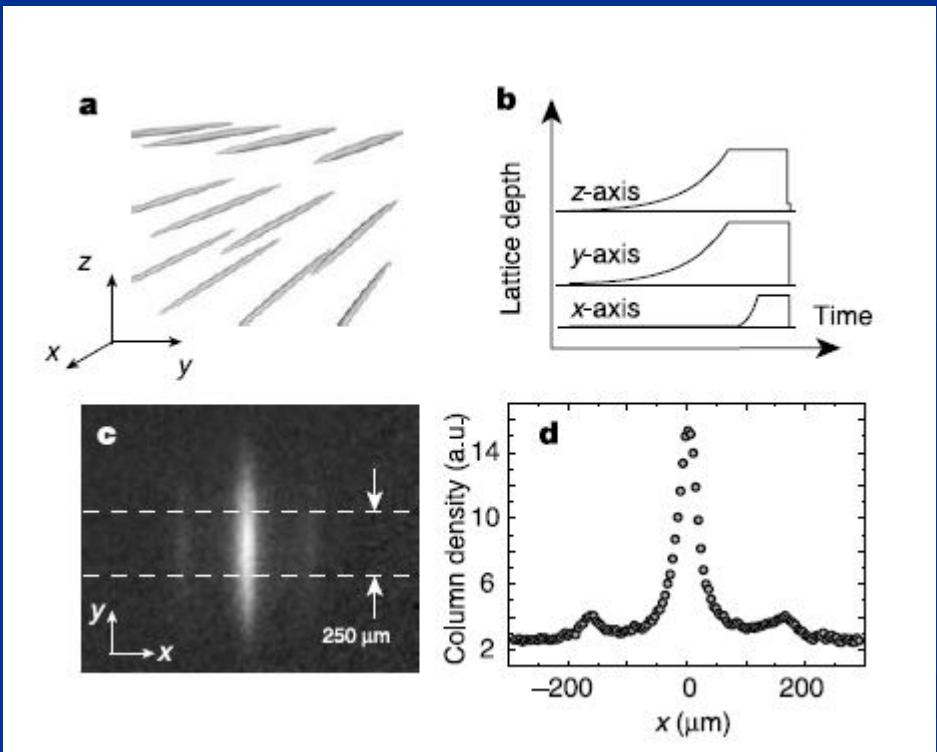
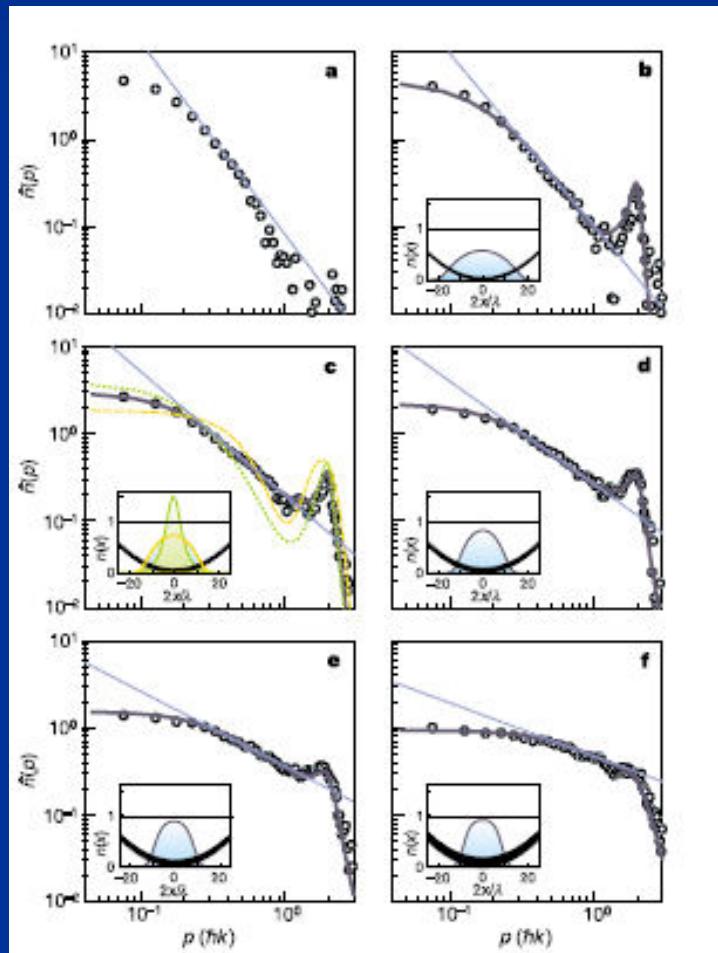
1125



Optical lattices (dilute)

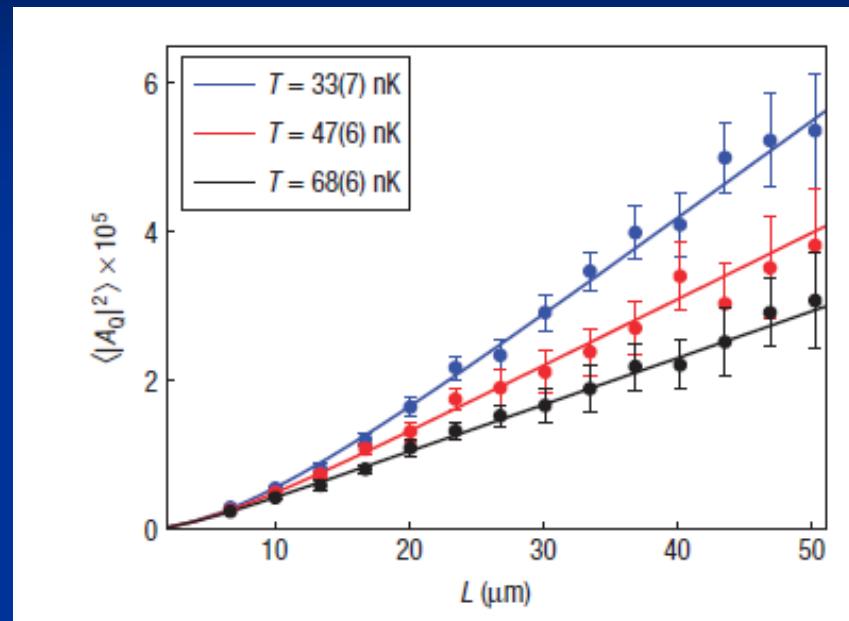
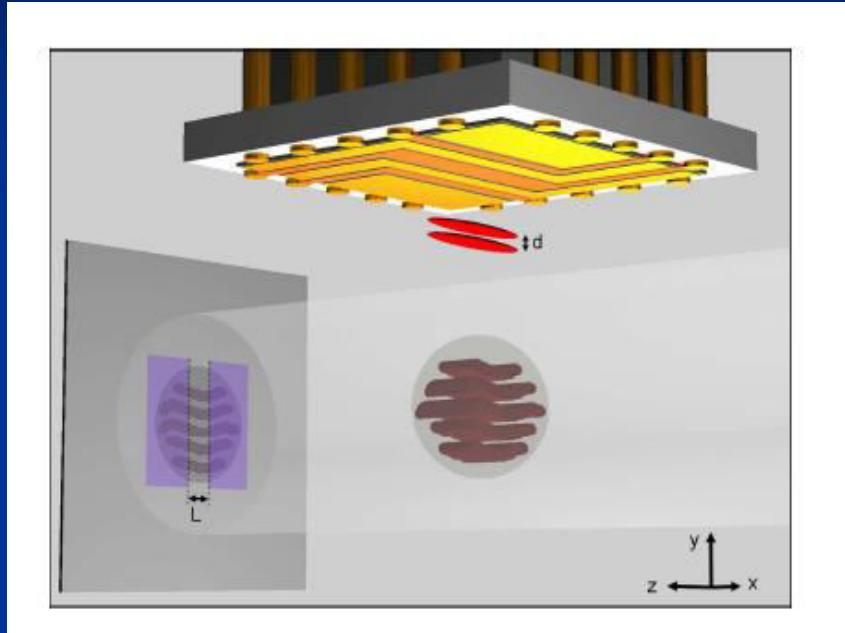


B. Paredes et al., Nature 429 277 (2004)



$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x) \psi(0) \rangle$$

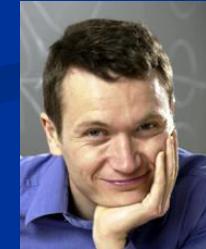
Atom chips



$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

K large (42)

S. Hofferberth et al. Nat. Phys 4
489 (2008)





Charge velocity

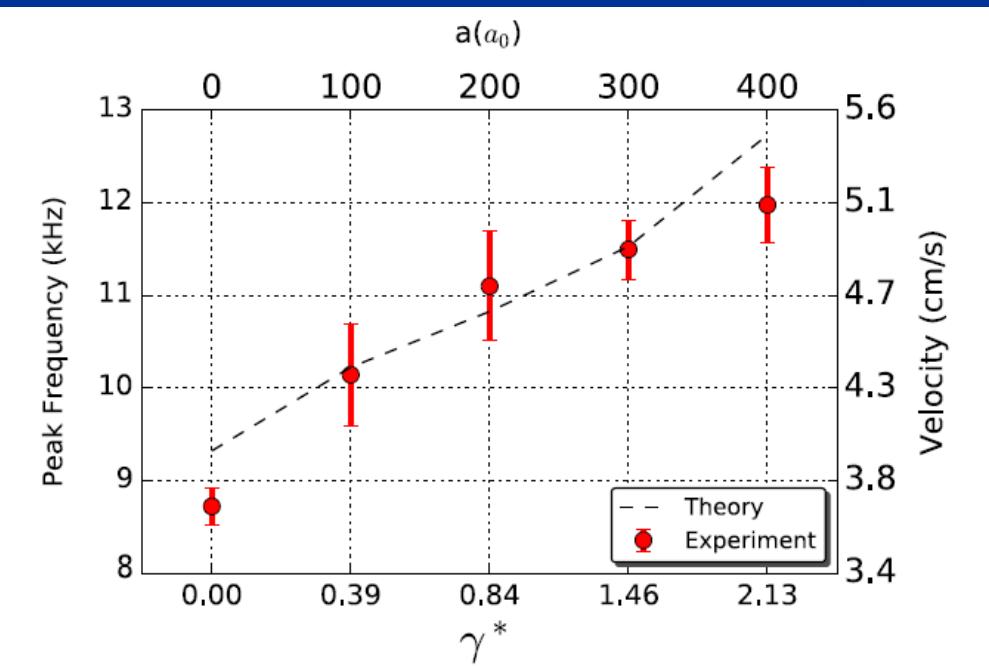
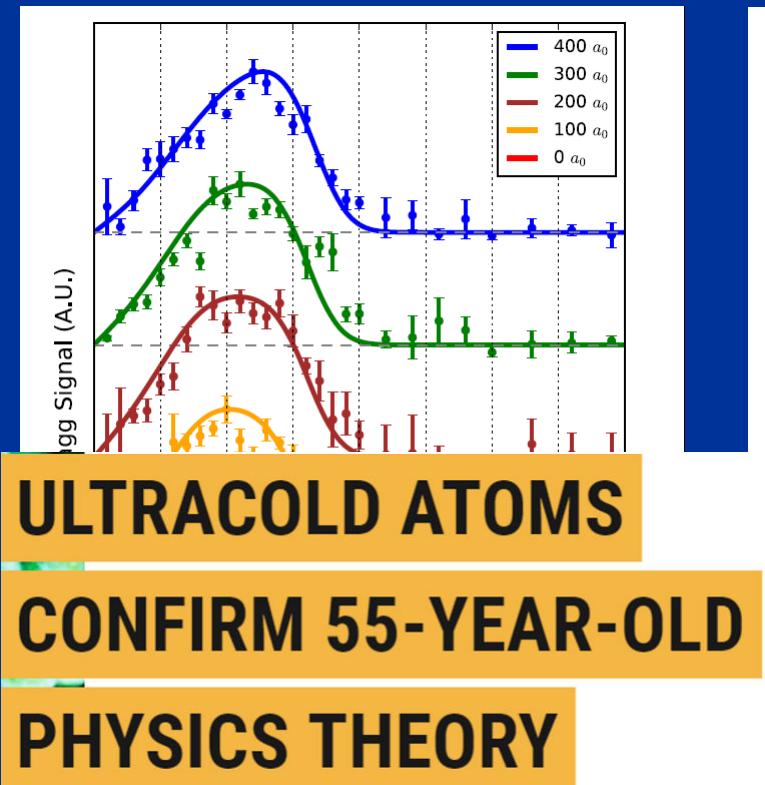


PHYSICAL REVIEW LETTERS 121, 103001 (2018)

Editors' Suggestion

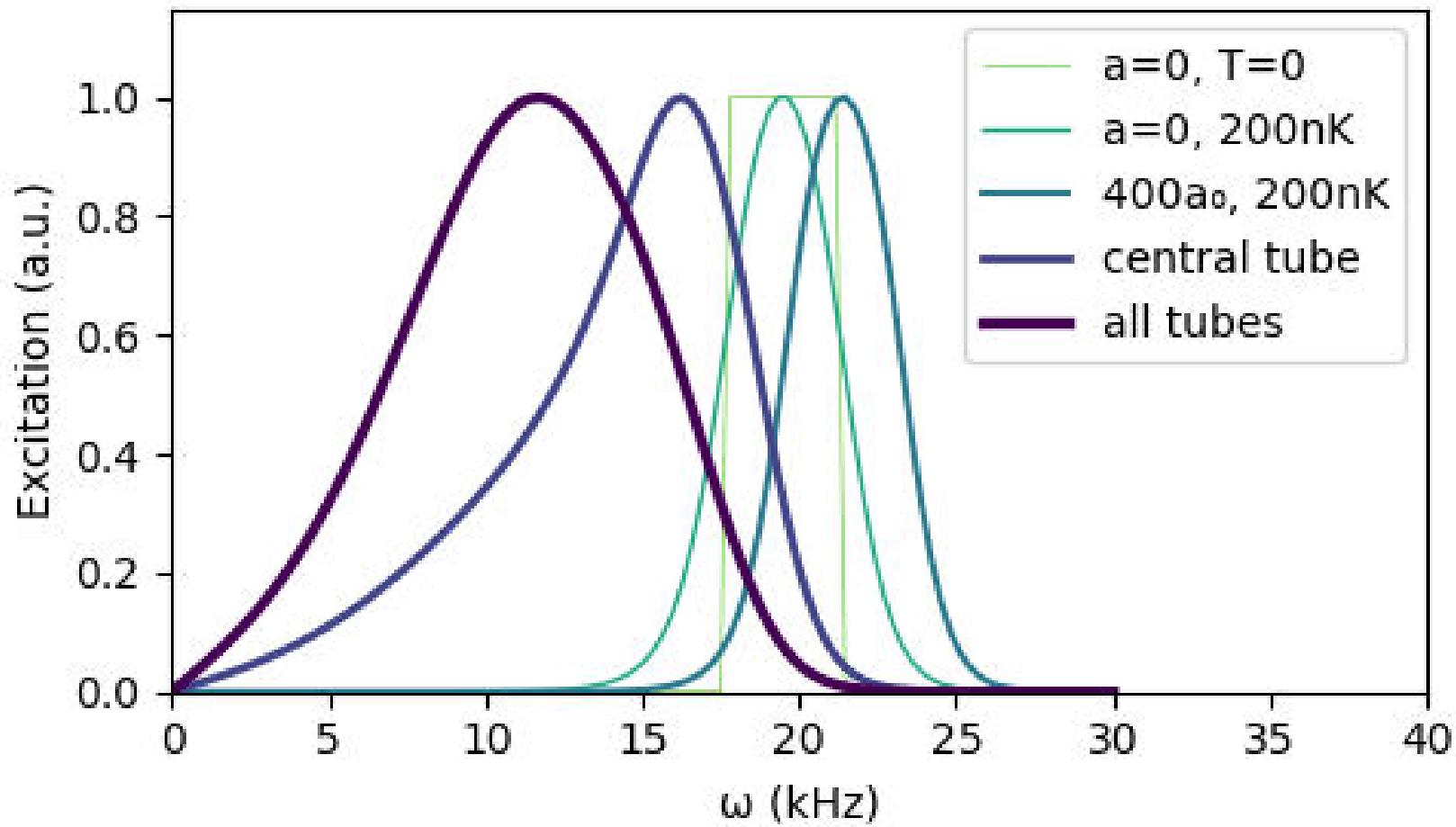
Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,¹ P. Grišins,² Y. T. Chang,¹ Z. H. Zhao,¹ C. Y. Shih,¹ T. Giamarchi,² and R. G. Hulet¹



<https://www.futurity.org/one-dimensional-electrons-physics-1858622/>

Effect of trap !



Fractionalization of excitations



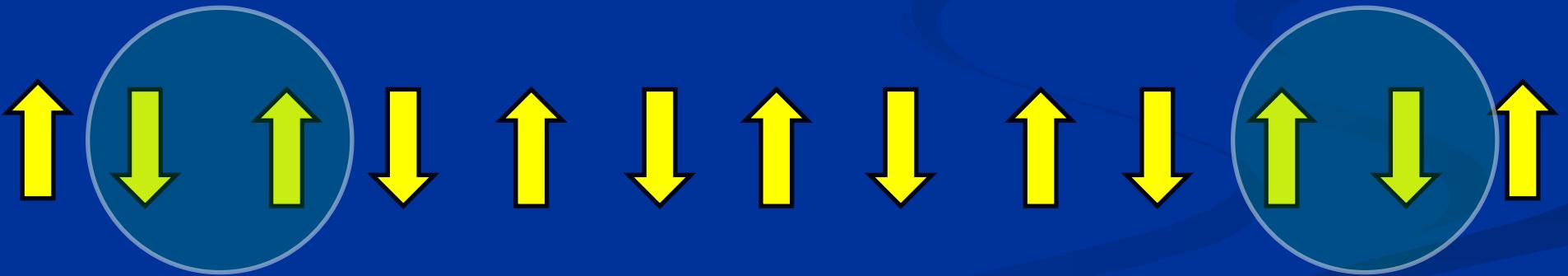
Fractionalization of excitations

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$



$$\Delta S = -1 \quad E = \delta(q)$$

Magnon



$$\Delta S = -1/2$$

Spinons

$$\Delta S = -1/2$$

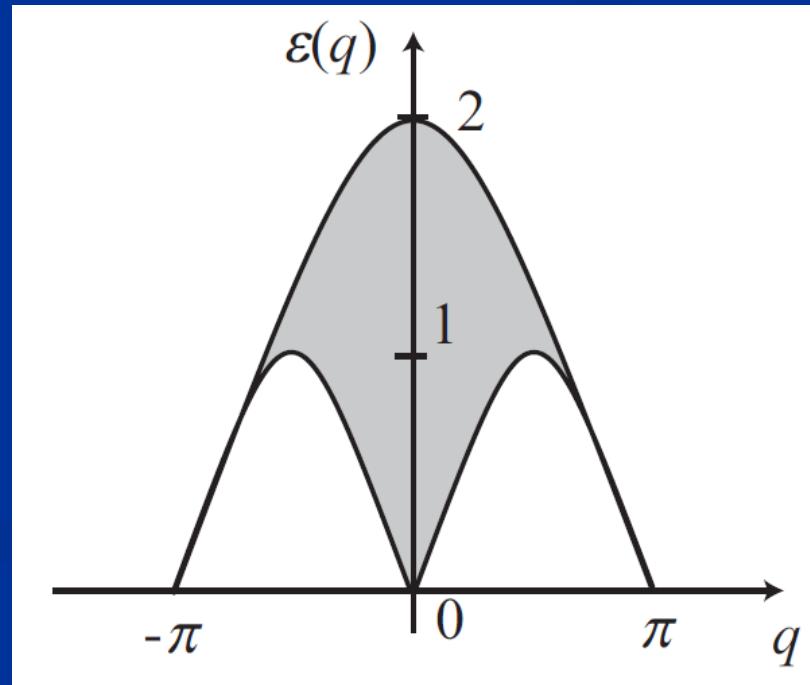
Magnons and spinons: $1 = \frac{1}{2} + \frac{1}{2}$



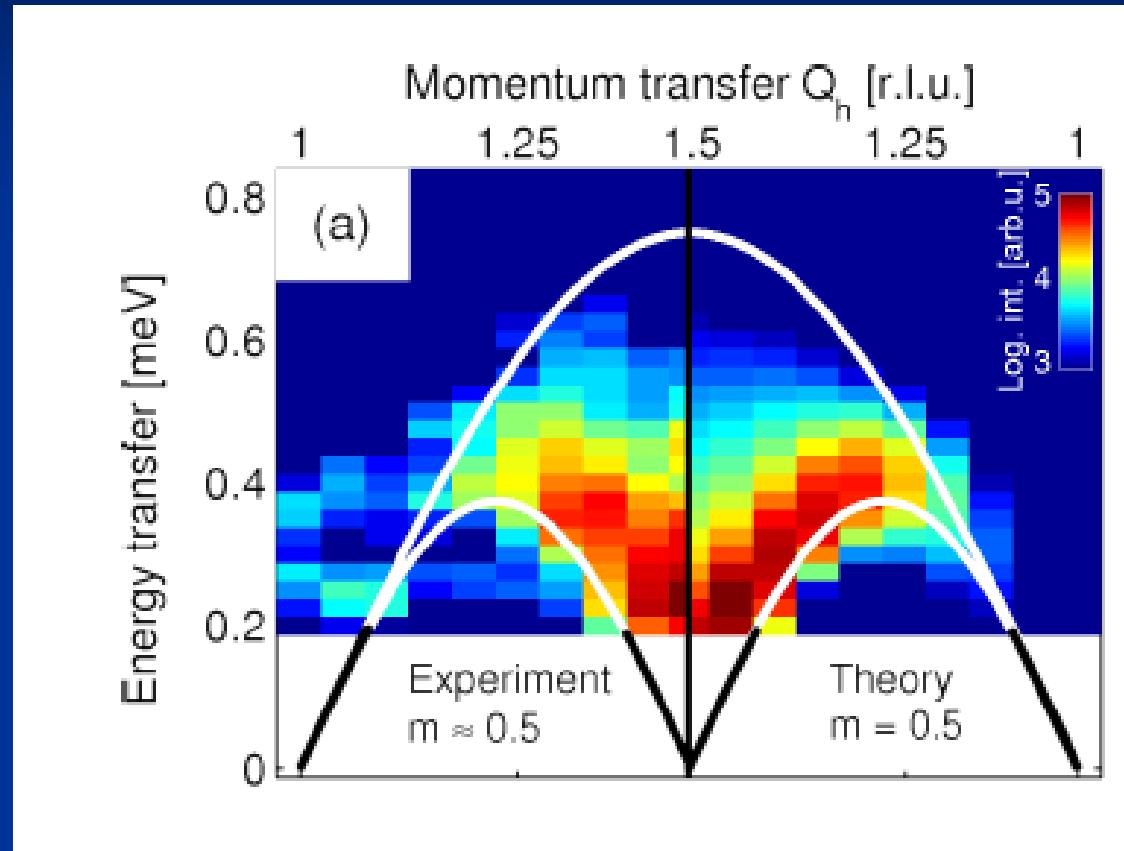
- Hidden (topological) order parameters
- Continuum of excitations

$$E(k) = \cos(k_1) + \cos(k_2)$$

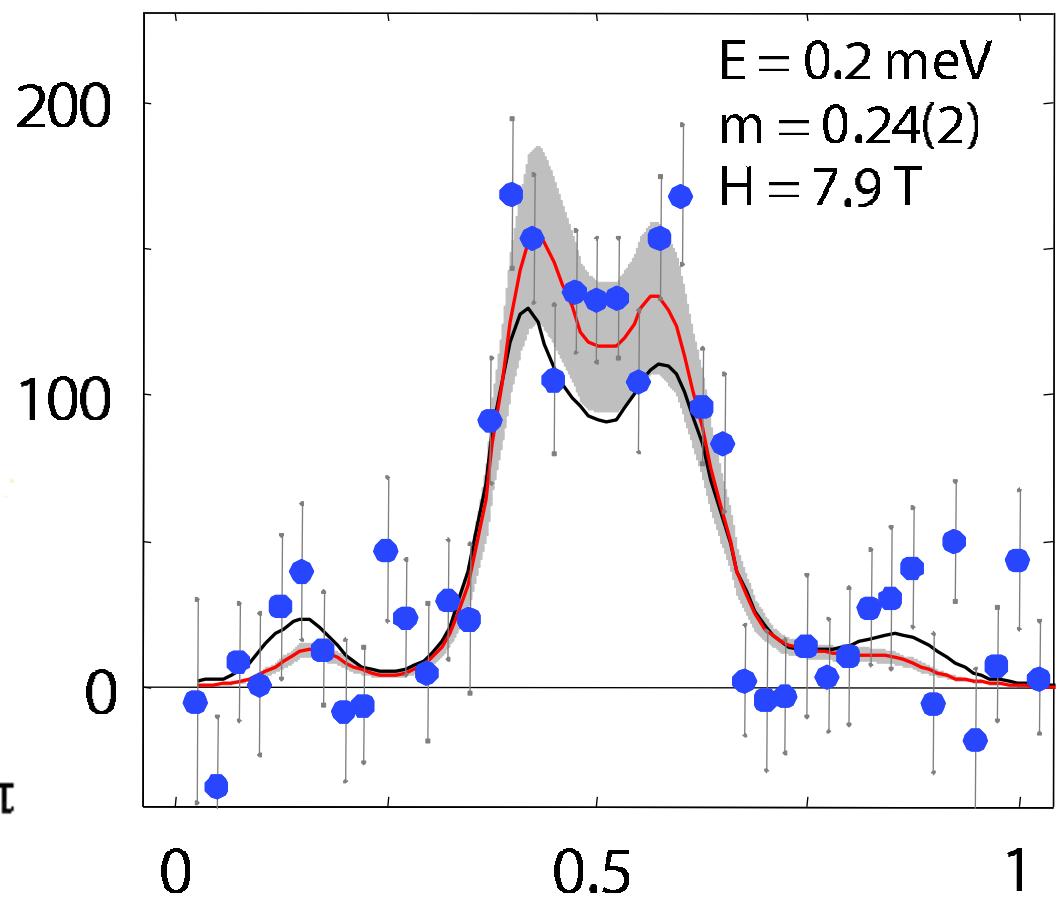
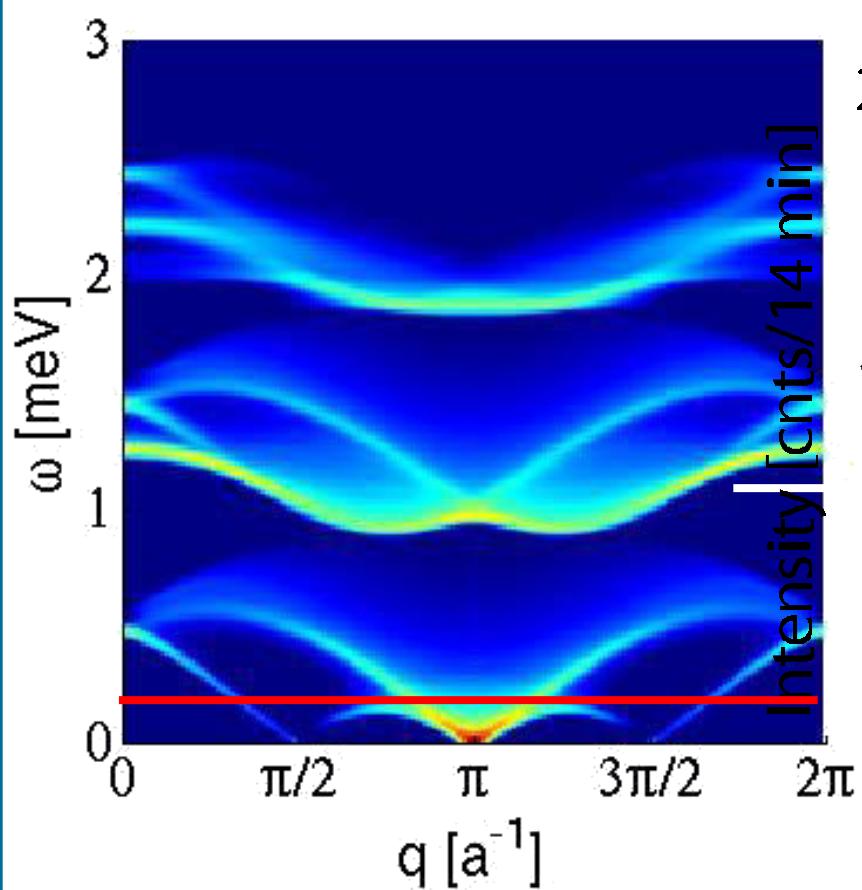
$$k = k_1 + k_2$$



Neutron scattering

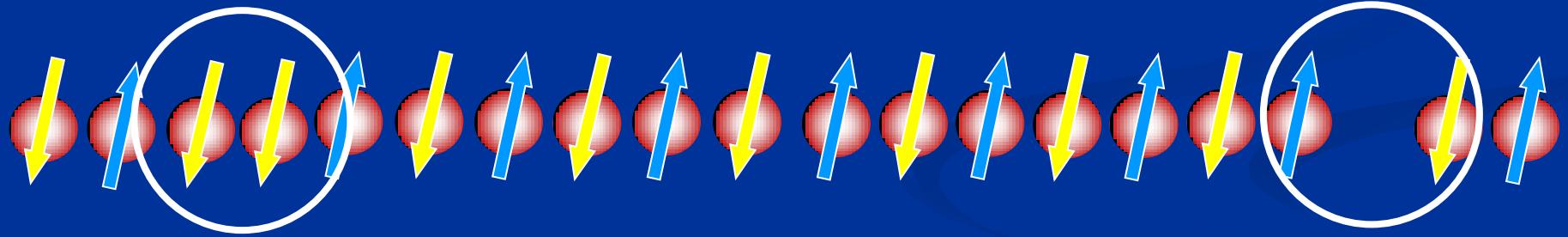


B. Thielemann et al. PRL 102, 107204 (2009)



Spin-Charge Separation

Spin



Spinon

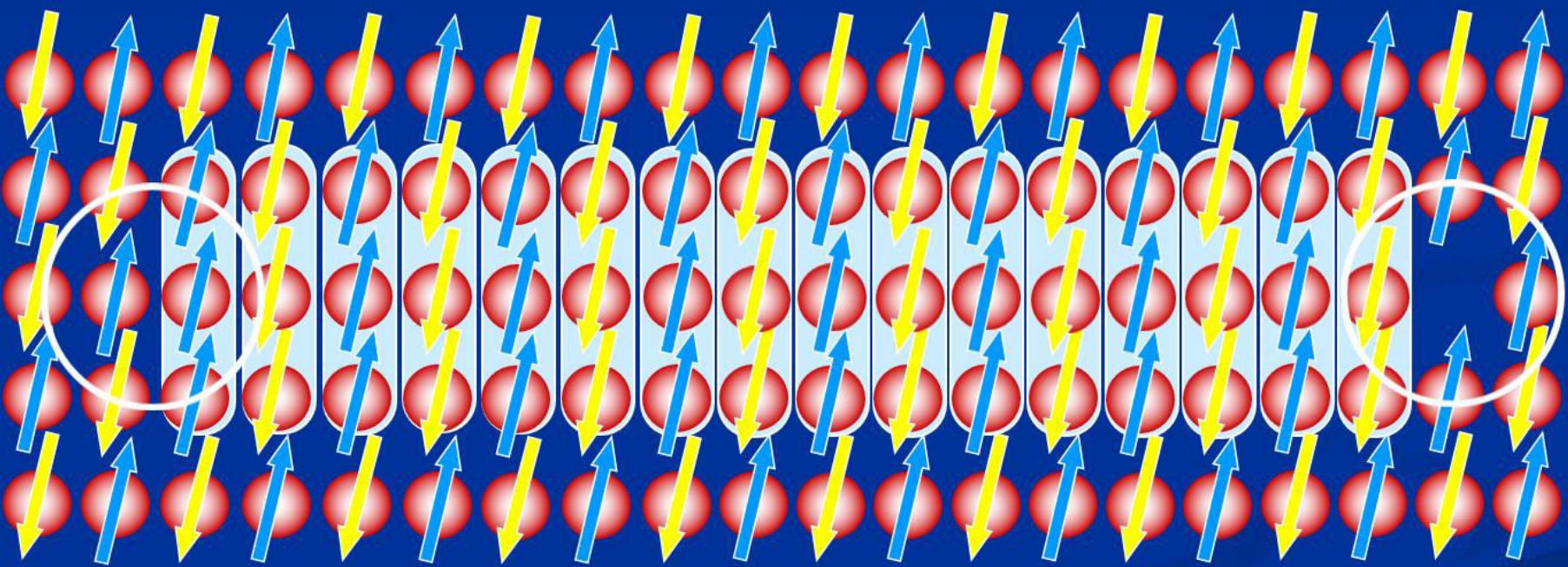
Charge

Holon

Spin-Charge Separation higher D ?

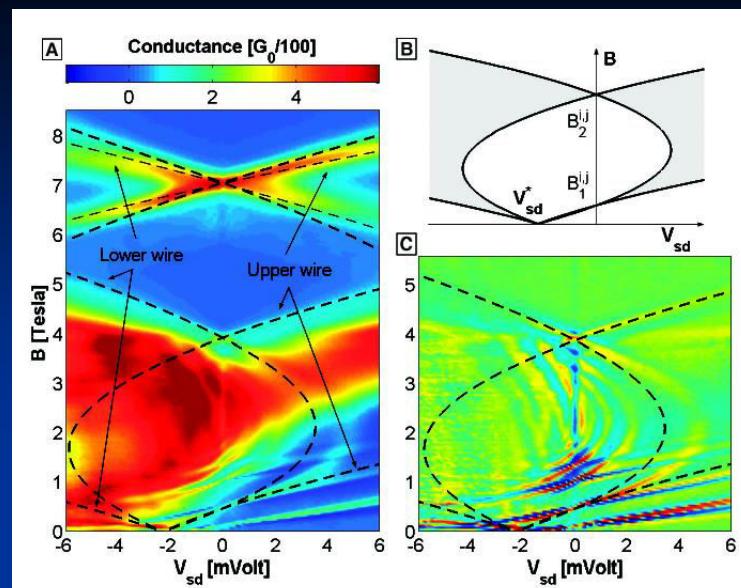
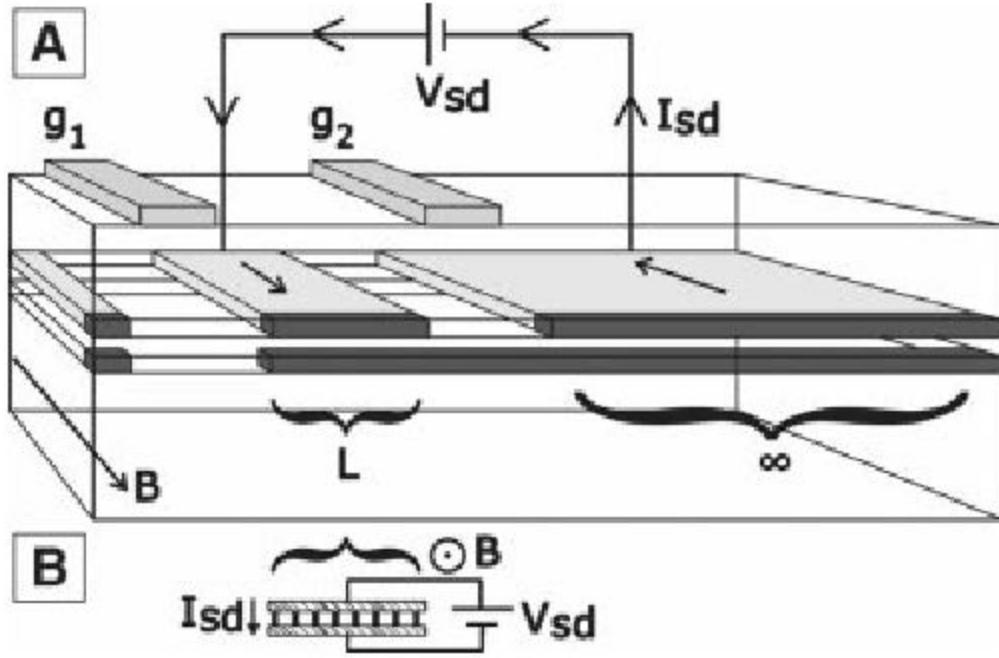
Spin

Charge



Energy increases with spin-charge separation

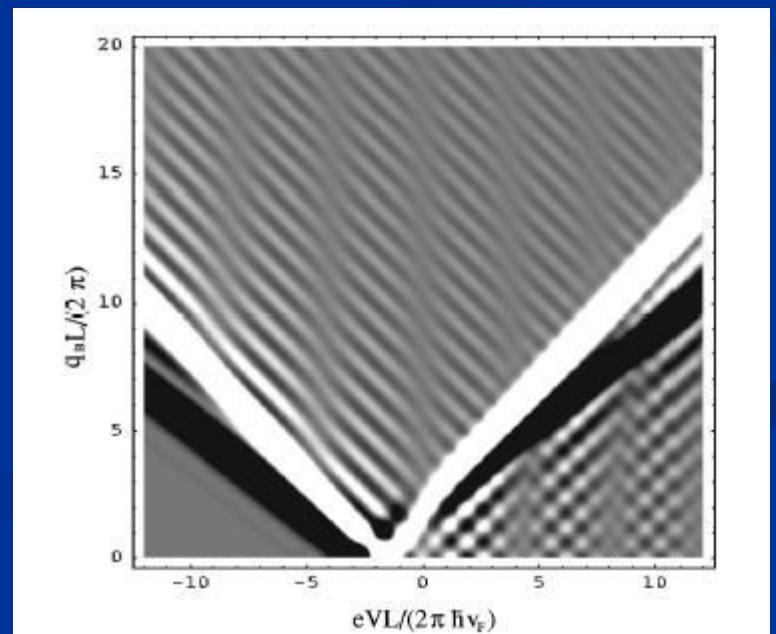
Confinement of spin-charge: « quasiparticle »



O.M Ausslander et al., Science
298 1354 (2001)

Y. Tserkovnyak et al., PRL 89
136805 (2002)

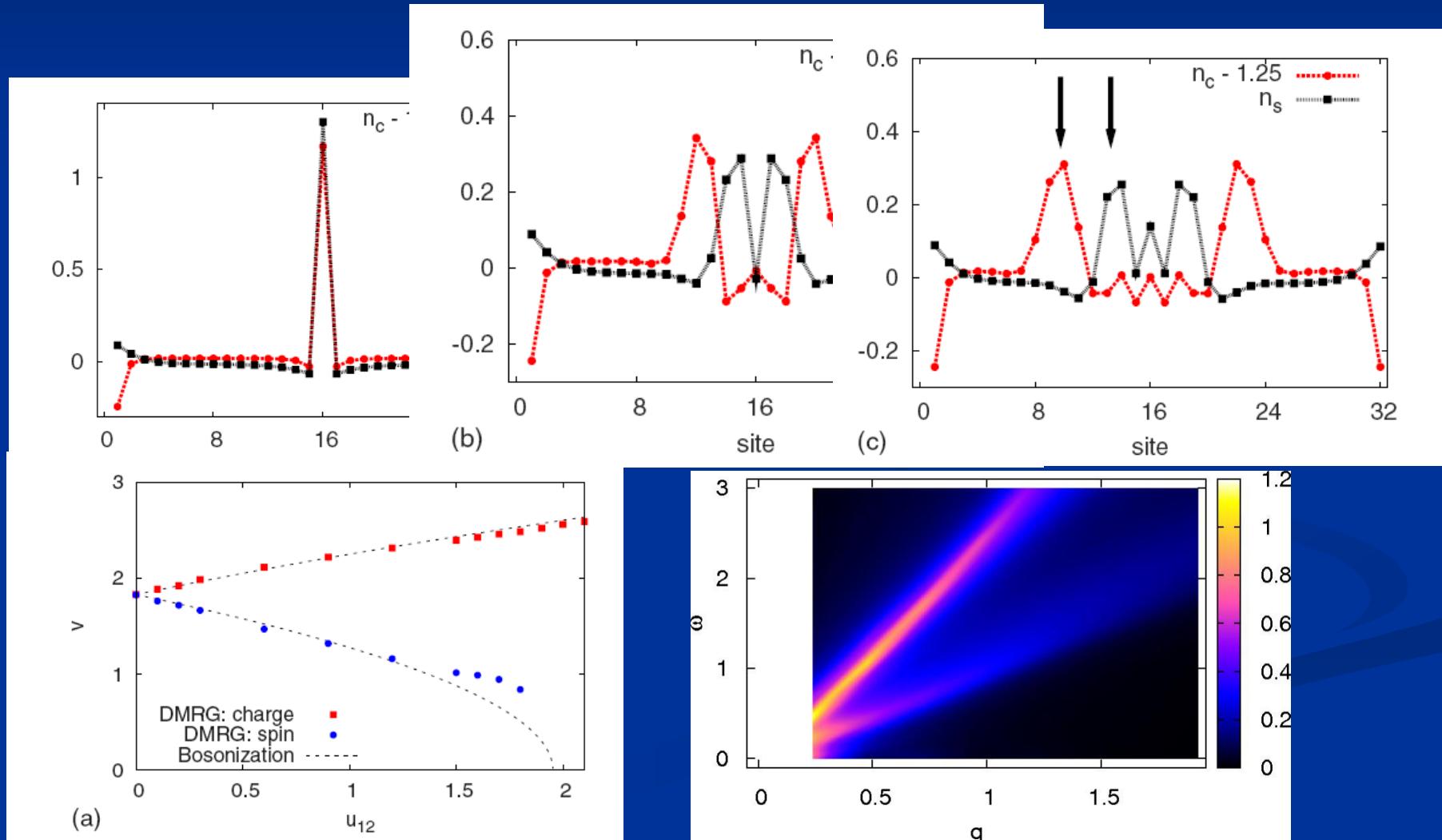
Y. Tserkovnyak et al., PRB 68
125312 (2003)



Proposal for cold atoms (Rb)



A. Kleine, C. Kollath et al. PRA 77 013607 (2008); NJP 10 045025 (2008)



Quantitative agreement with the predictions of the Luttinger liquid

End of the story ??????

NO! Luttinger liquid plays the same role than Fermi liquid did

To boldly go where no theorist
has gone before....

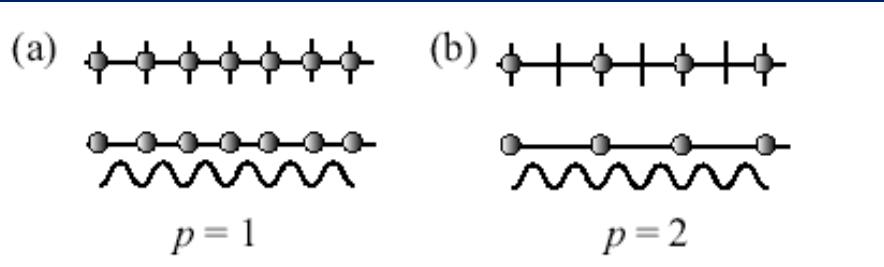


Mott transition



How to treat?

TG, Physica B
230 975 (97)



$$H = \int dx V_0 \cos(Qx) \rho(x)$$

$$H = \int dx V_0 \cos(Qx) \rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2 \pi \rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

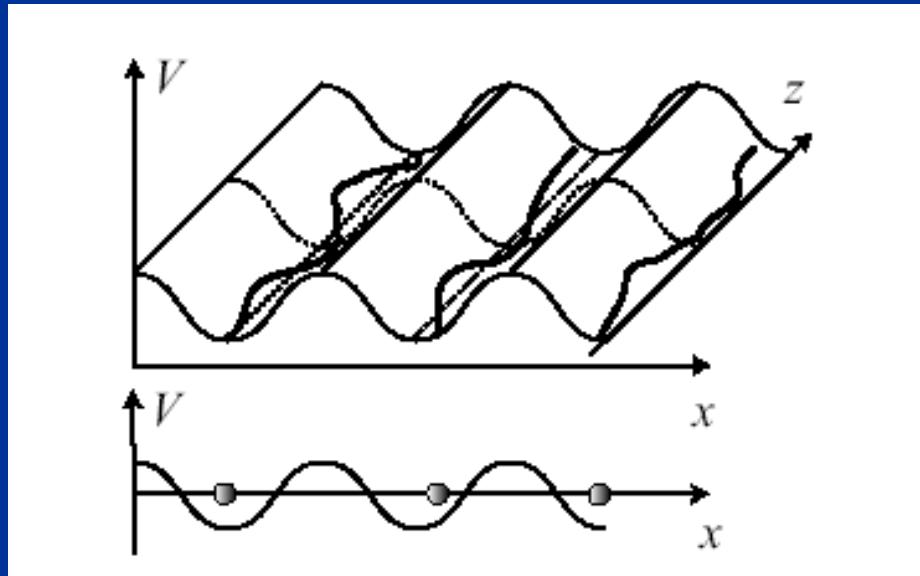
- Commensurate: $Q = 2 \pi \rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

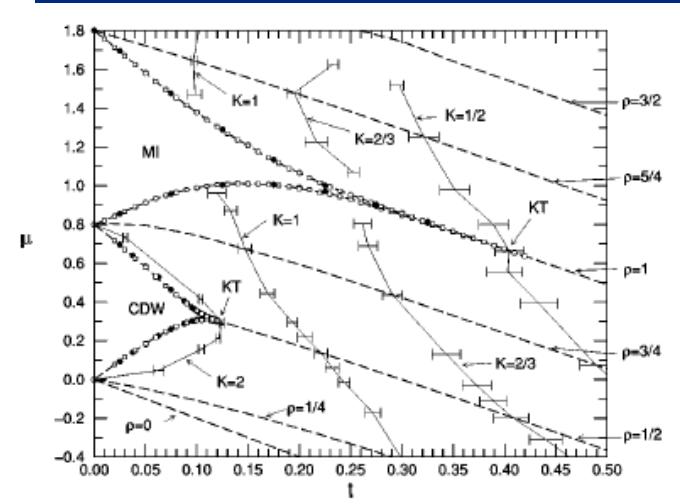
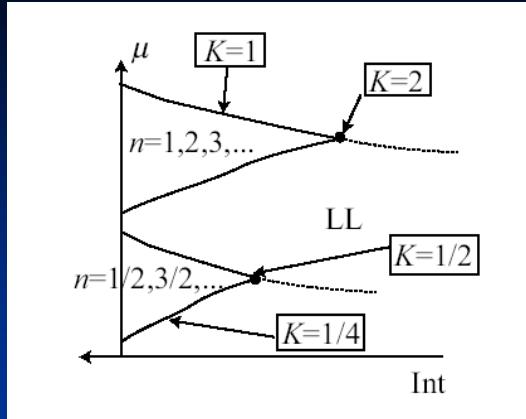
$$S_0 = \int \frac{dxd\tau}{2\pi K} [\frac{1}{u} (\partial_\tau \varphi(x, \tau))^2 + u (\partial_x \varphi(x, \tau))^2]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\phi(x))$$



Beresinskii-
Kosterlitz-Thouless
transition at K=2

String order
parameter



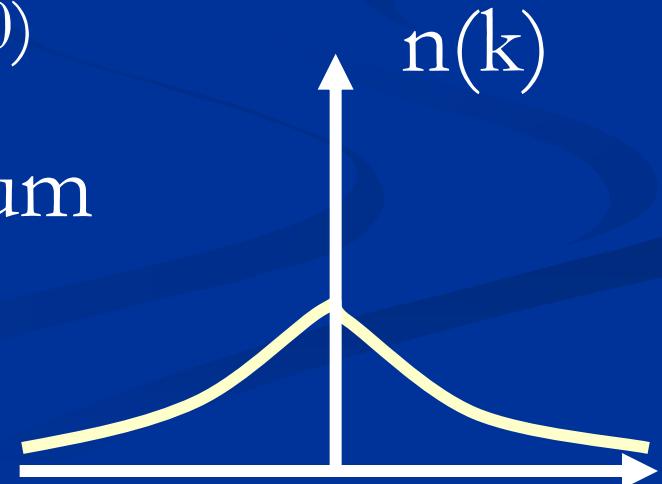
T. Kuhner et al. PRB 61 12474 (2000)

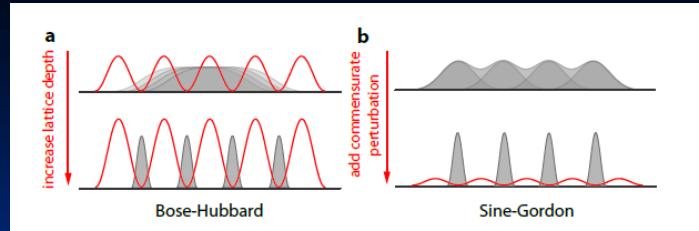
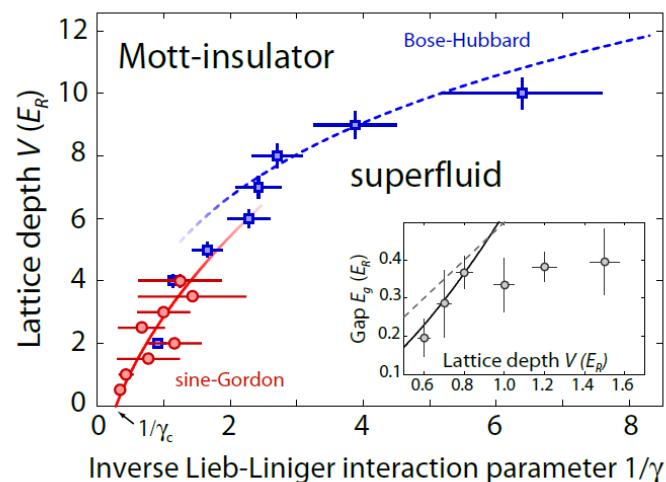
Gap in the excitation spectrum

$$G(r) \propto e^{-r/\xi}$$

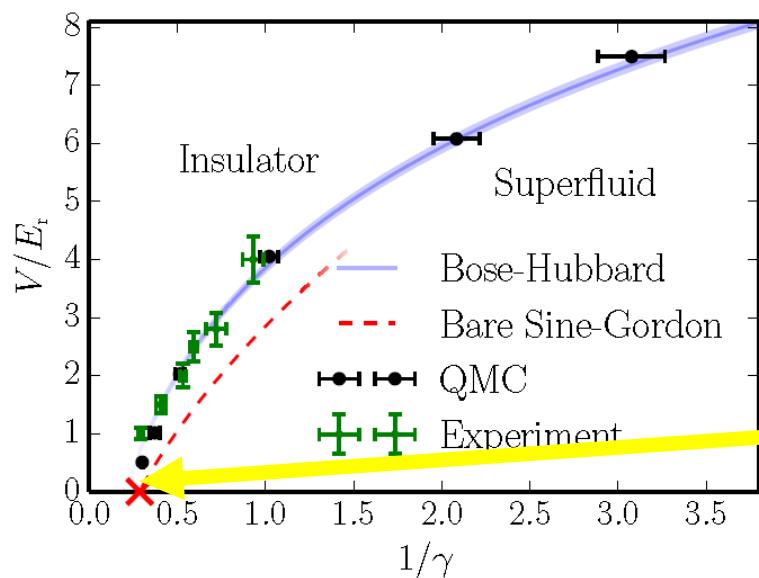
Mott insulator:
 ϕ is locked
 Density is fixed

TG, Physica B
 230 975 (97)





E. Haller et al. Nature 466 597 (2010)



Renormalized
Sine-Gordon

Shows:
 $K^* = 2$

G. Boeris et al. PRA 93 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla \phi(x)$$

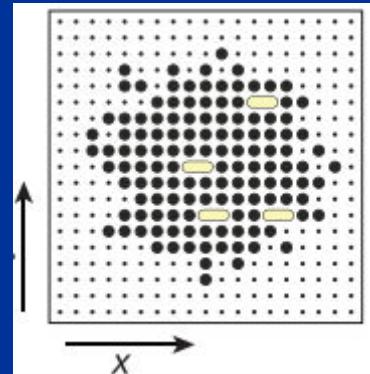
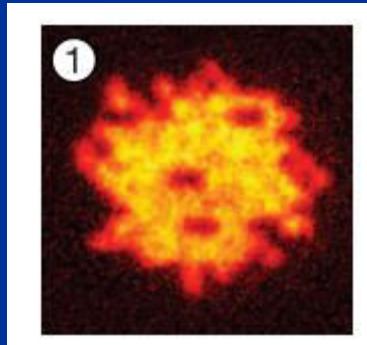
$$\mathcal{O}_P^2 = \lim_{l \rightarrow \infty} \mathcal{O}_P^2(l) = \lim_{l \rightarrow \infty} \left\langle \prod_{k \leq j \leq k+l} e^{i\pi \delta n_j} \right\rangle$$

E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman,
Phys. Rev. B **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

Science (2011)

M. Endres,^{1*} M. Cheneau,¹ T. Fukuhara,¹ C. Weitenberg,¹ P. Schauß,¹ C. Gross,¹ L. Mazza,¹
M. C. Bañuls,¹ L. Pollet,² I. Bloch,^{1,3} S. Kuhr^{1,4}



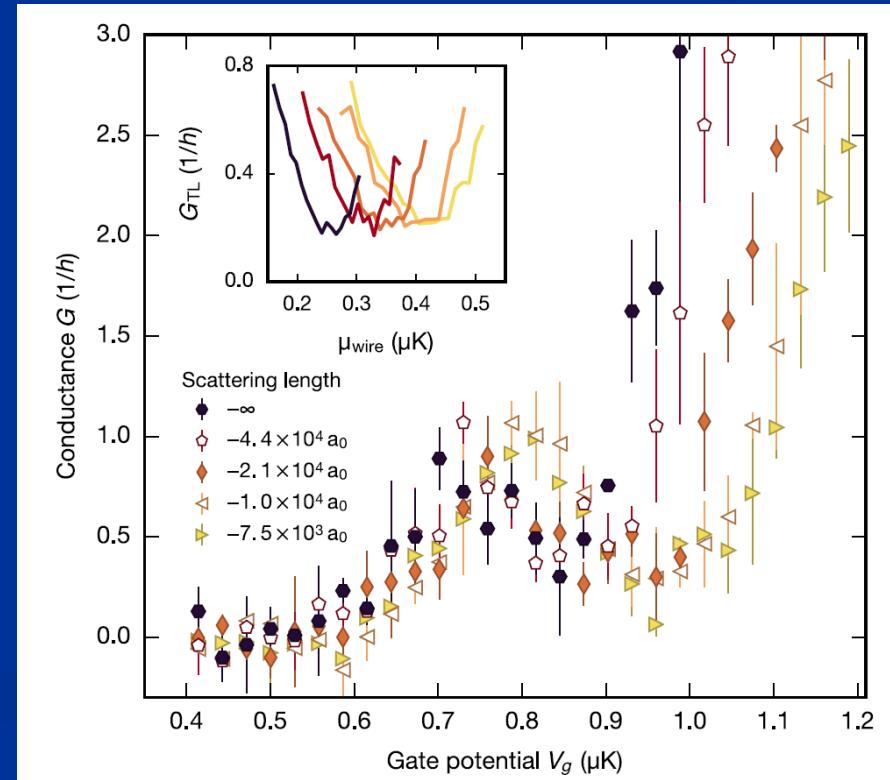
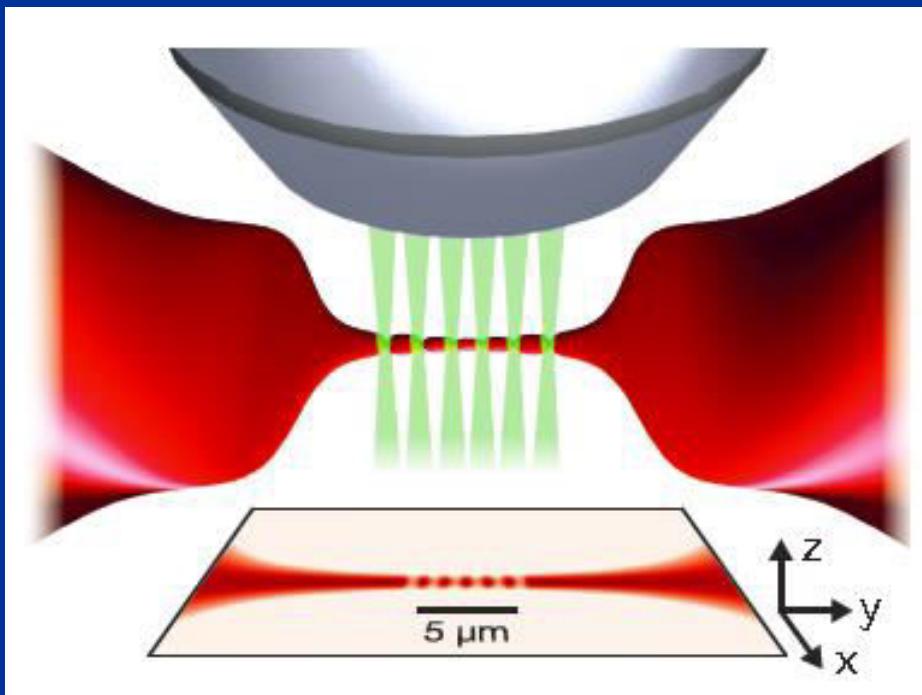
Beyond Luttinger liquids

- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures,
- New type of quantum critical points (e.g. topological)...
- Transport; Out of equilibrium situations.



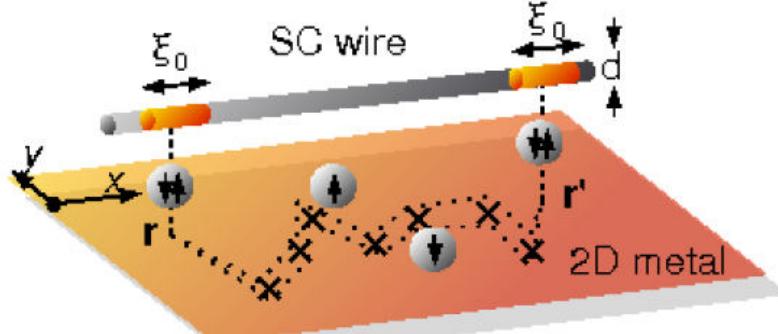
Quantum transport in 1D

M. Lebrat, P. Grisins et al., PRX 8 011053 (2018)

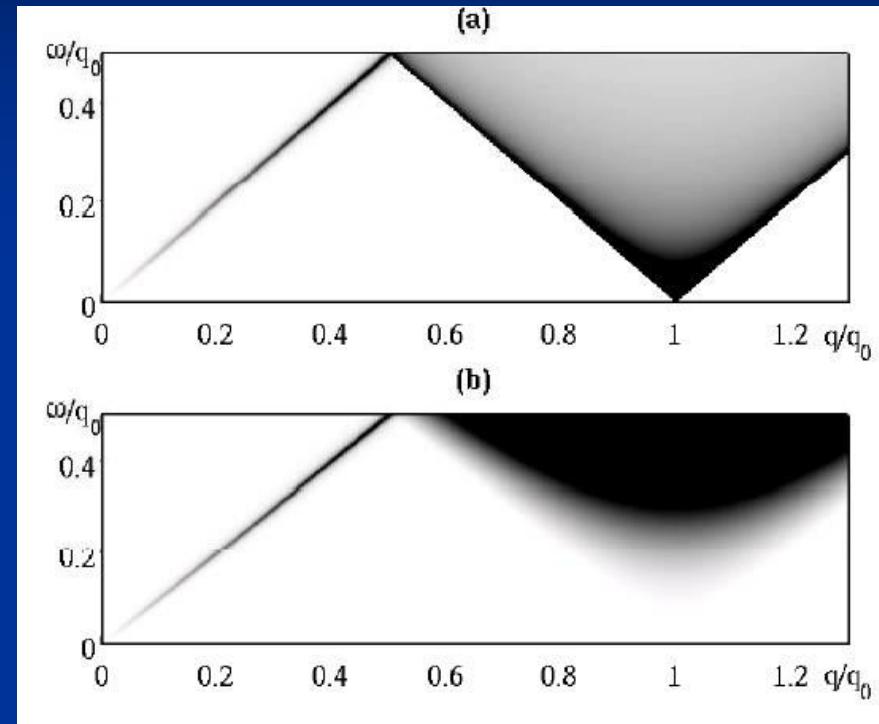


Coupled 1D chains: deconfinement



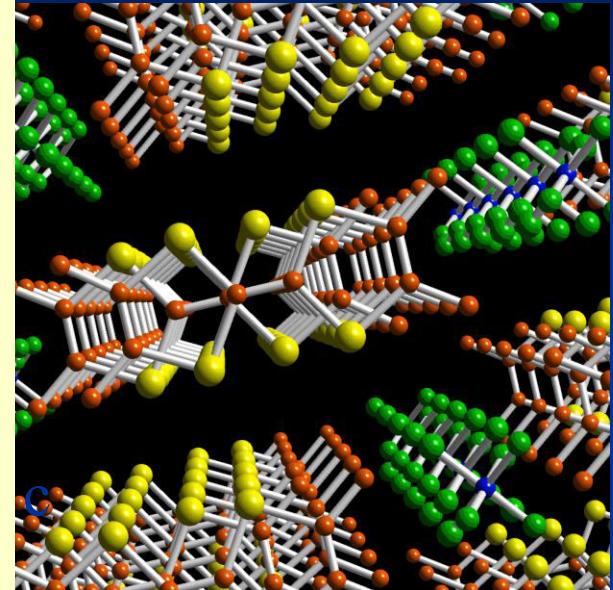
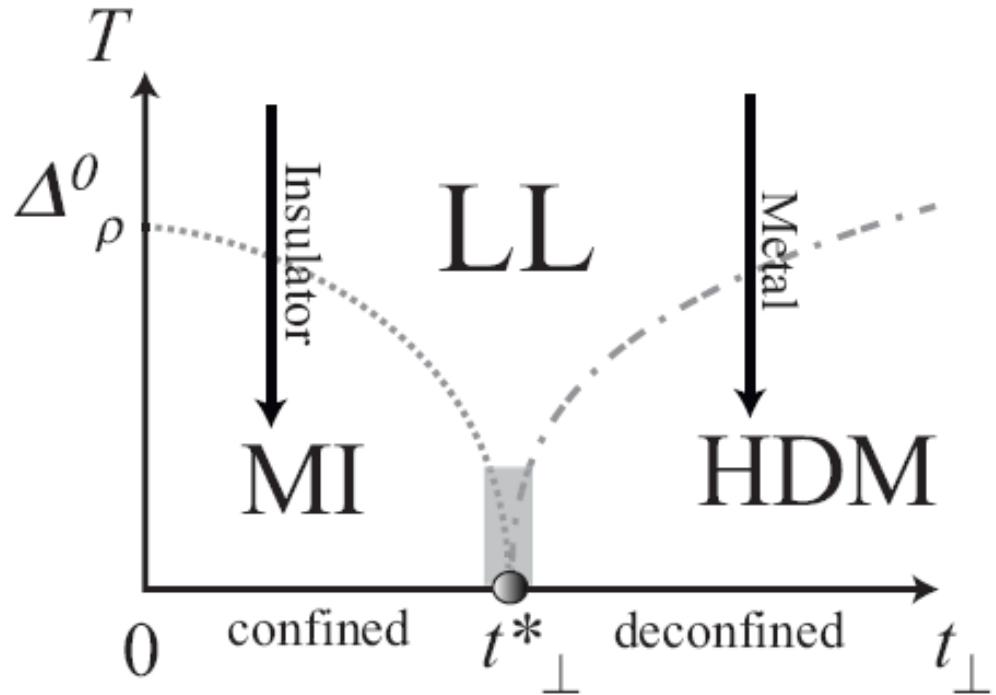


A. Lobos, A. Iucci, M. Muler,
TG PRB 80 214515 (09)



E. Dalla Torre, E. Demler, TG,
E. Altman, Nat. Phys. 6 806 (2010)

Deconfinement

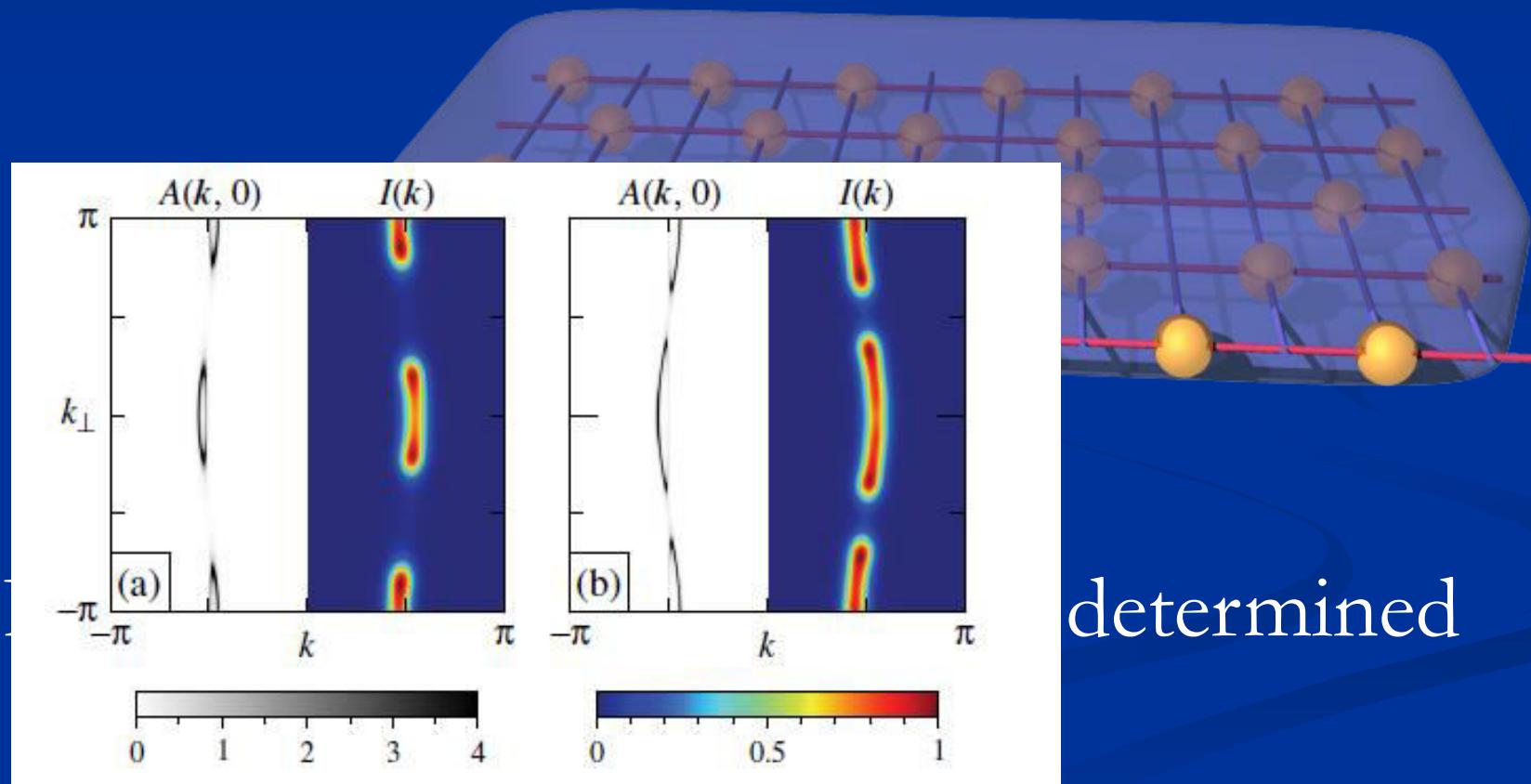


TG Chemical
Review 104 5037
(2004)

P. Auban-Senzier, D. Jérôme, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, D. MacCannell, M.J. Hanfland, G.A. Koutscher, PRB 81 125109 (2010)

Back to the (self-consistent) bath

S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)



Some other problems

- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures,
- Out of equilibrium situations
 - A. Mitra, TG, Phys. Rev. Lett. 107, 150602 (2011)
 - E. Dalla Torre, E. Demler, TG, E. Altman, Nat. Phys. 6 806 (2010); PRB 85 184302 (2012)

- Ladders and magnetic fields:
E. Orignac, TG Phys. Rev. B **64**, 144515 (2001)
M. Atala et al., arxiv/1402.0819
- Impurities, polarons in 1D systems:
- Dimensional crossover 1d – 2d/3d:
AF Ho, M.A. Cazalilla, TG, PRL **92** 130403
(2004);NJP **8** 158 (2006).

And many other problems
and works.....



(A.M. Visuri, N. Kamar, S. Greshner, F. Hartmeier, C. Bardyn, M. Filippone, C. Berthod, T. Pellegrin, N. Kestin, S. Takayoshi)

Spin systems: A. Tsvelik, P. Bouillot, C. Kollath, ...

Exp Groups: C. Berthier, B. Grenier, C. Ruegg, A. Zheludev

Cold atoms: M. Cazalilla, A. Ho, M. Zvonarev, V. Cheianov, U. Schollwoeck, P. Torma, ...

Exp Groups: G. Modugno, M. Inguscio, S. Kuhr, I. Bloch, T. Esslinger, J.P. Brantut

Conclusions

- Tour of one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful and challenging questions going beyond the Luttinger liquids
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Many experimental realizations both in condensed matter and in cold atoms

