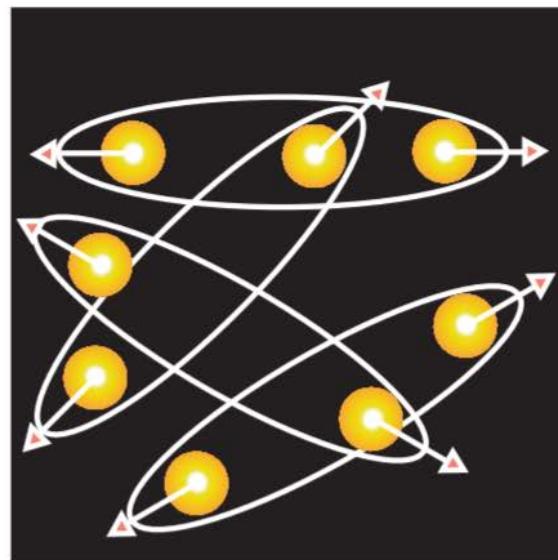


Attractive fermions: the BEC- BCS crossover and the unitary gas (II)

Frédéric Chevy

Les Houches 2018 School on Ultracold Fermions

The (3D) BEC-BCS crossover with cold atoms

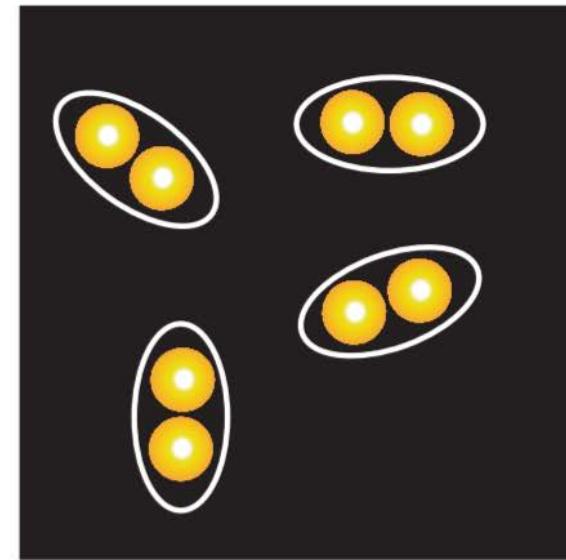


BCS: weakly attractive



$1/k_F a$

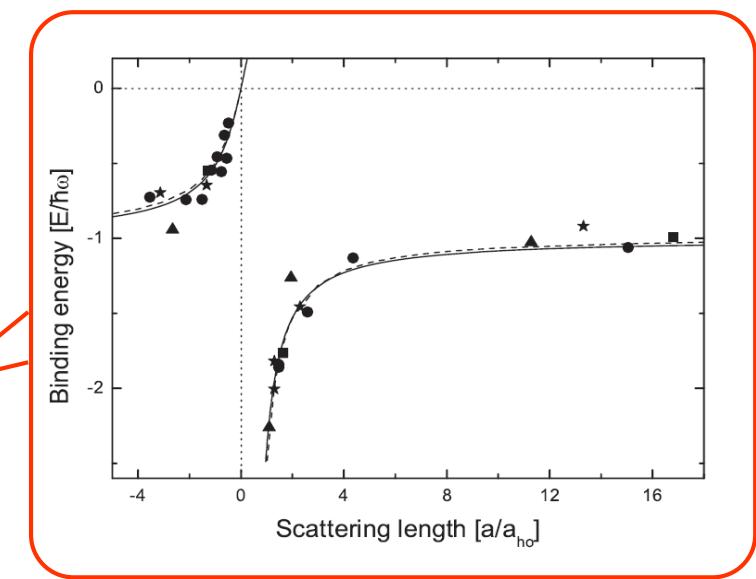
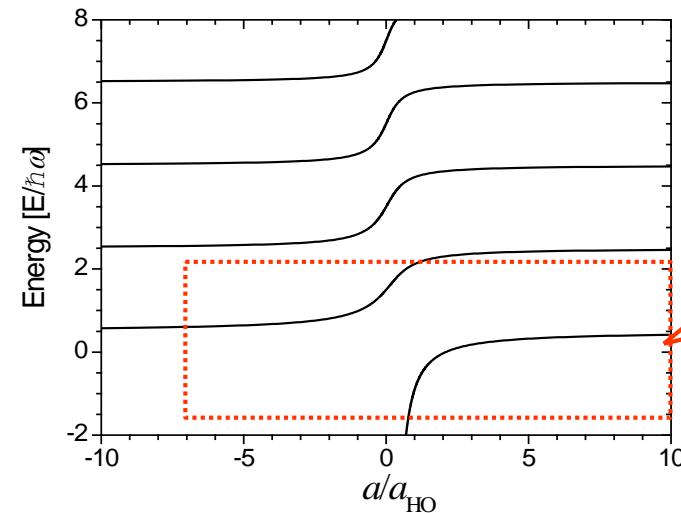
BEC: strongly attractive



Upper-branch vs lower branch physics

- For a mean-field BEC: $\mu = gn \Rightarrow$ attractive for $g<0$, repulsive for $g>0$.
BUT The atomic BEC and the BEC of dimers do not live in the same branch of solutions.

Toy model: 2 atoms in a harmonic potential

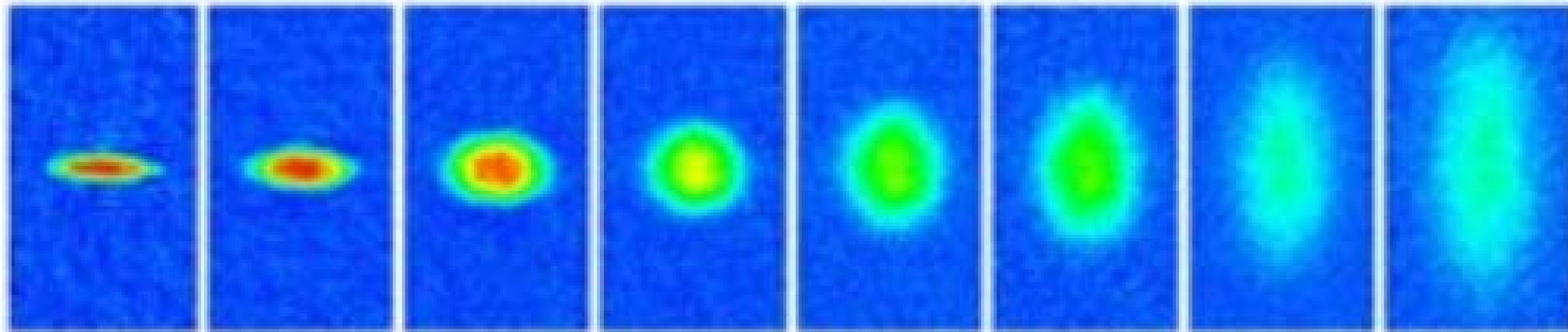


Probing superfluidity

Is the Fermi superfluid a superfluid?

Classical tests of superfluidity in ultracold BECs

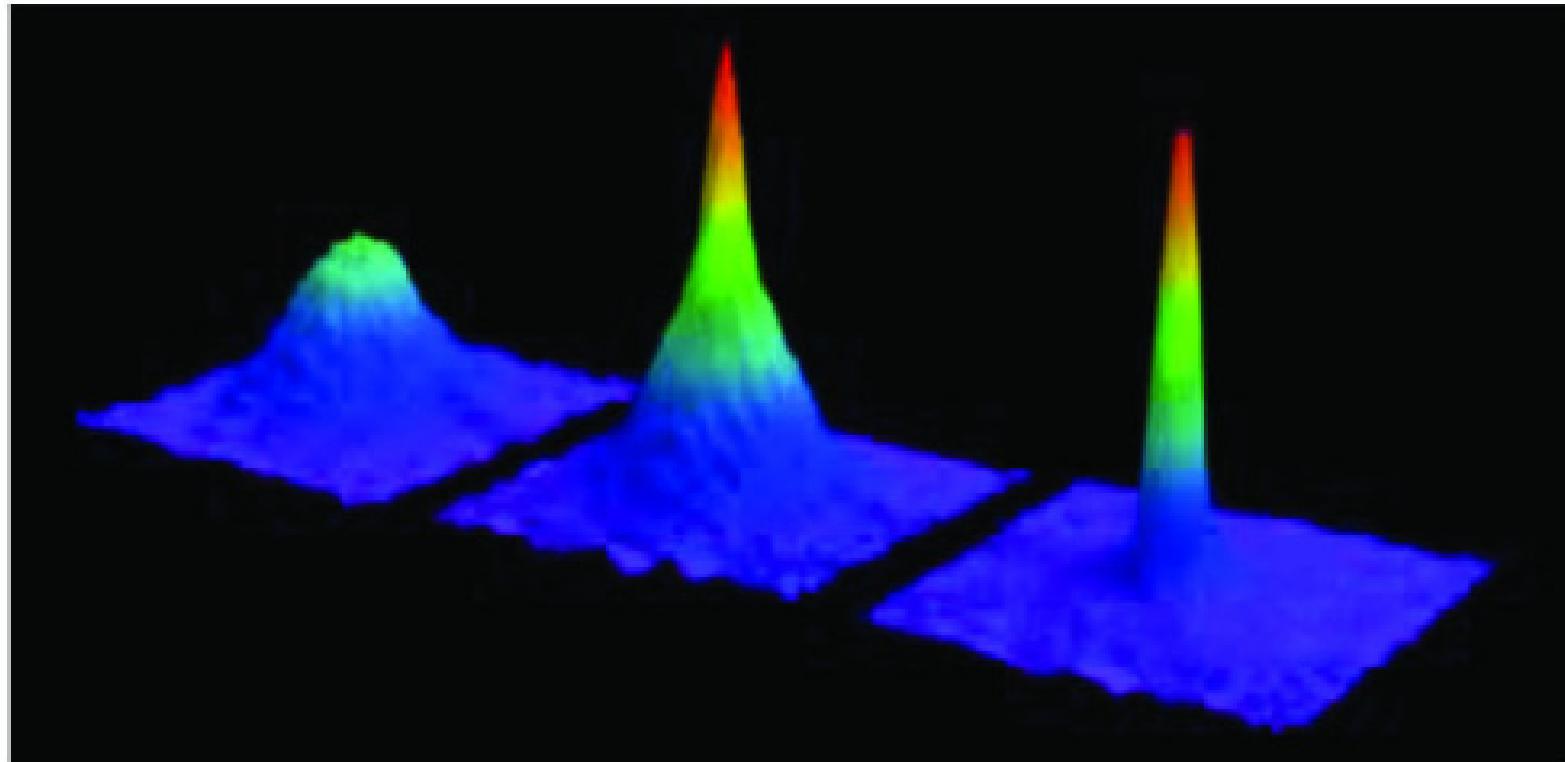
- ~~Inversion of the ellipticity in time of flight~~ Proves hydrodynamics
- Double structure of the momentum distribution
- Vortices



Is the Fermi superfluid a superfluid?

Classical tests of superfluidity in ultracold BECs

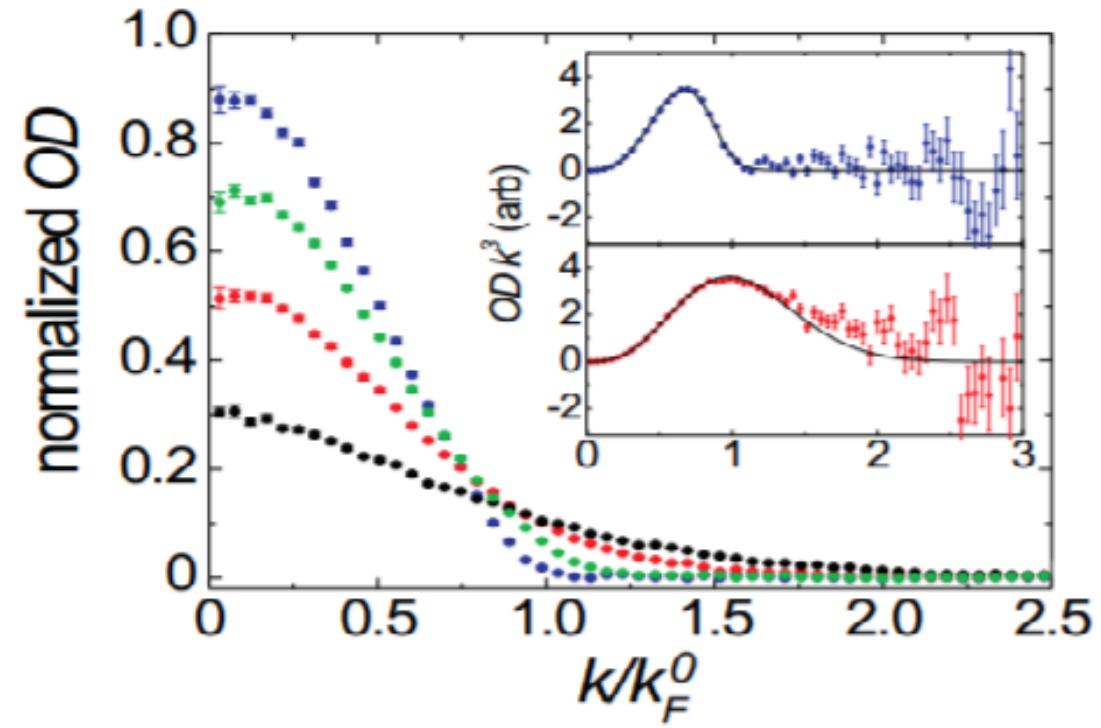
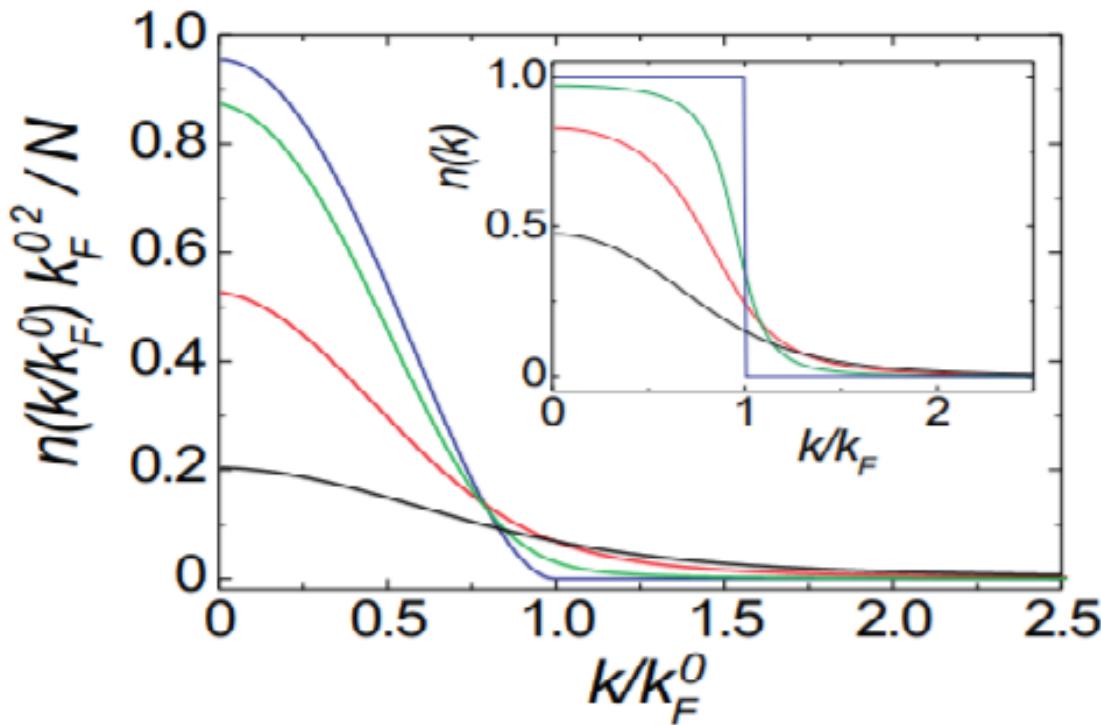
- ~~Inversion of the ellipticity in time of flight~~ Proves hydrodynamics
- ~~Double structure of the momentum distribution~~
- Vortices



Is the Fermi superfluid a superfluid?

Classical tests or superfluidity in ultracold BECs

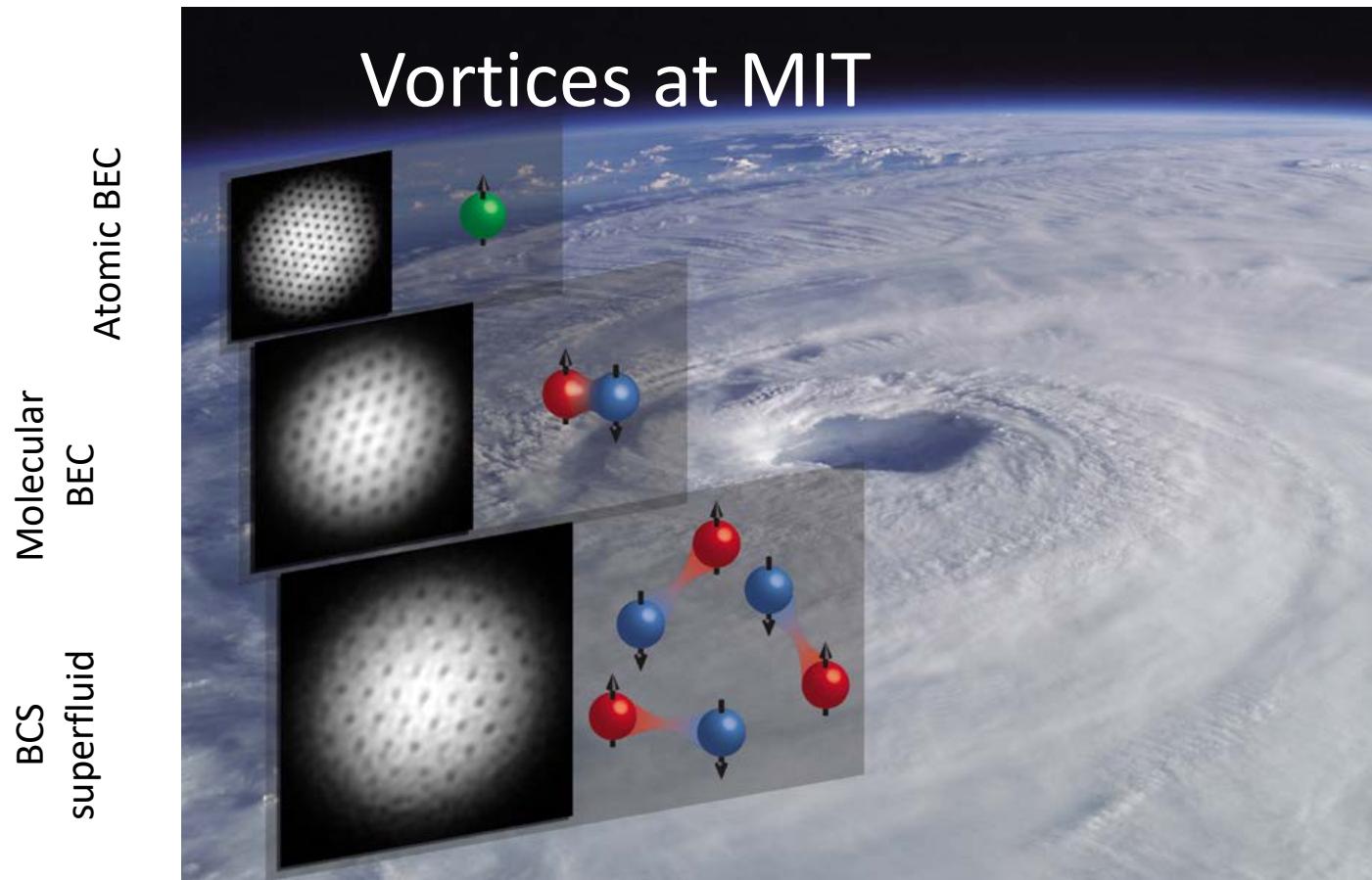
- ~~Inversion of the ellipticity in time of flight~~ Proves hydrodynamics
- ~~Double structure of the momentum distribution~~
- Vortices



Is the Fermi superfluid a superfluid?

Classical tests or superfluidity in ultracold BECs

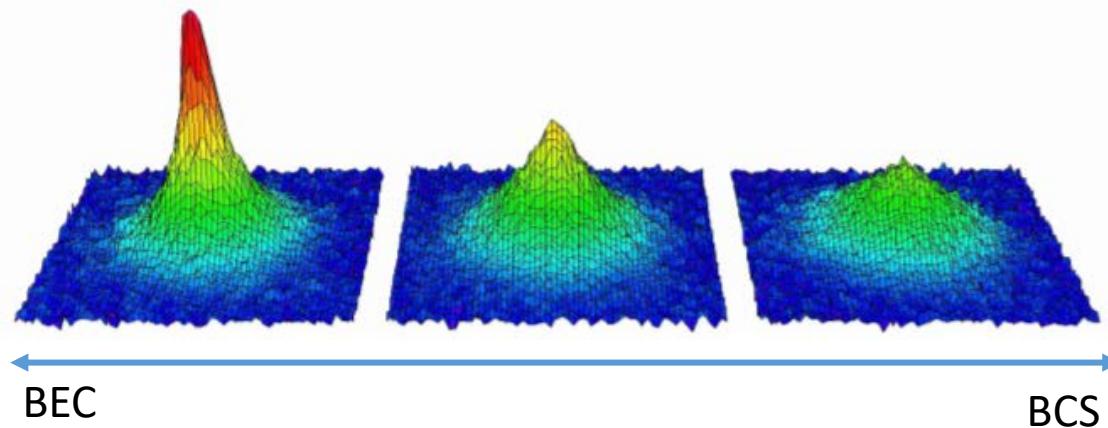
- ~~Inversion of the ellipticity in time of flight~~ Proves hydrodynamics
- ~~Double structure of the momentum distribution~~
- Vortices



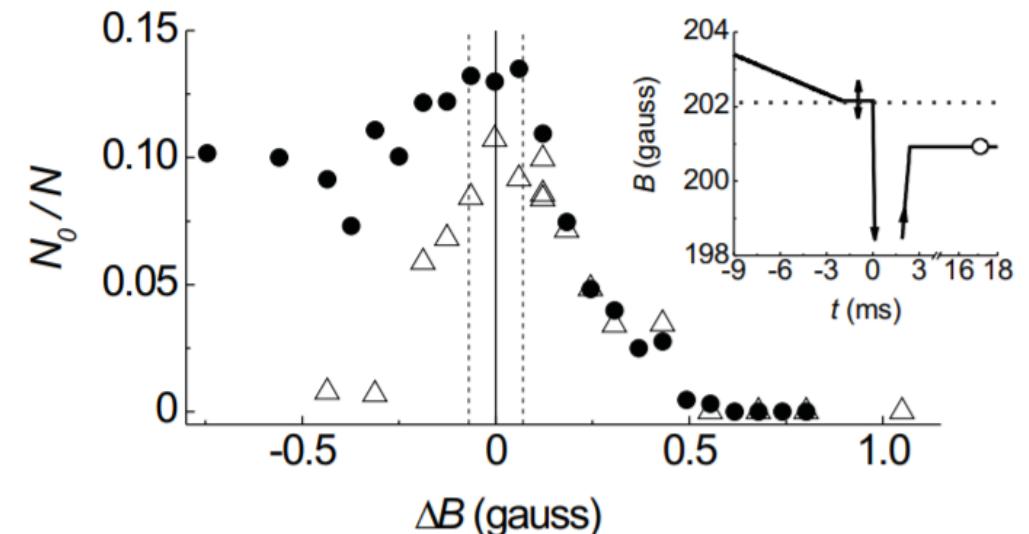
The pair projection method

Idea: salvage the time of flight technique by sweeping the magnetic field to the BEC side of the resonance to project Cooper pairs onto molecules and observe the double structure (MIT, JILA 2004)

Trick: go fast enough to prevent rethermalization but not too fast to avoid breaking the pairs.



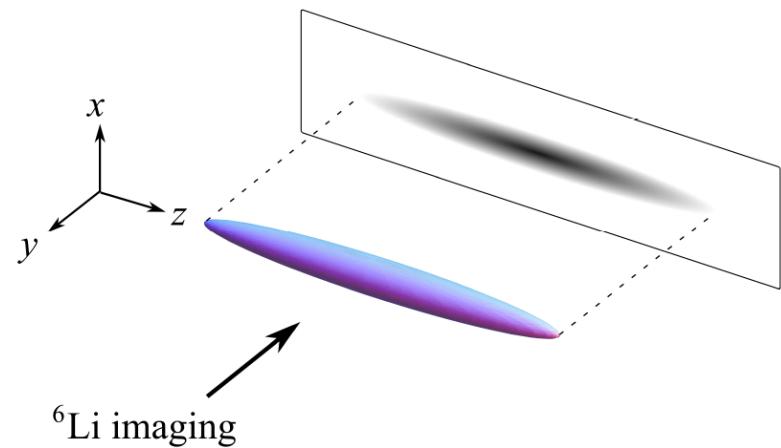
Regal *et al.* PRL 2004.



Thermodynamics of a strongly correlated Fermi gas

Zero-temperature equation of state

Absorption imaging



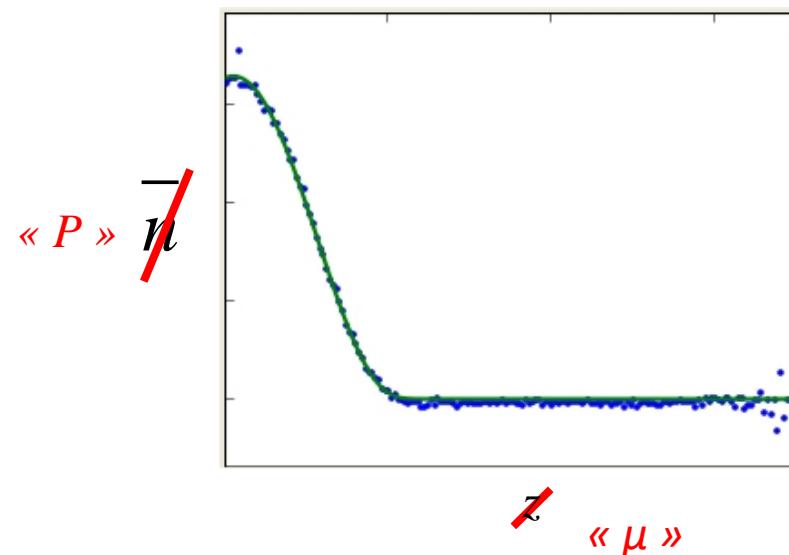
High accuracy (few percent):

- Double integration increases S/N ratio
- One shot yields a whole piece of the Equation of state

$$\bar{n}(z) = \int dx dy n(x, y, z)$$

In a harmonic trap (T.L. Ho & Q. Zhou - Nature Physics (2010))

$$\bar{n}(z) = \frac{2\pi P(\mu - m\omega_z^2 z^2, T)}{m\omega_{\perp}^2}$$



Equation of State in the BEC-BCS crossover

Goal: calculate the equation of state of the gas as a function of $1/k_F a$

Dimensionally:

$$E = \frac{N}{2} E_b + \frac{3NE_F}{5} \xi(1/k_F a)$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} \quad k_F = (3\pi^2 n)^{2/3}$$

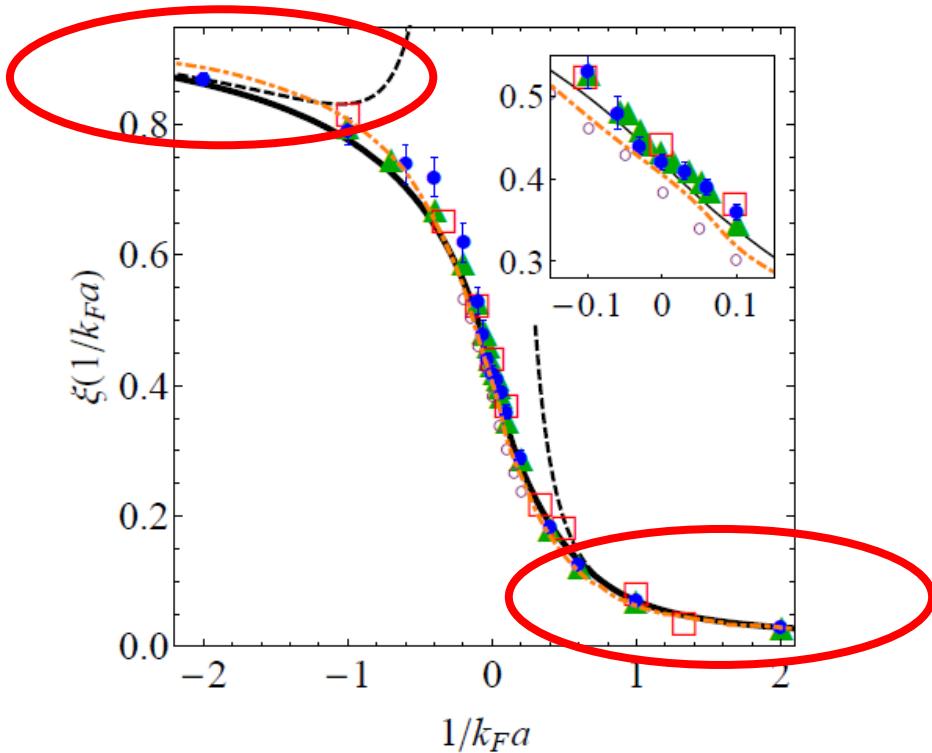
$$E_b = -\frac{\hbar^2}{ma^2} \Theta(a)$$

Asymptotic behavior:

$1/k_F a \rightarrow 0^-$ (BCS Limit): weakly attractive Fermi gas, $\xi \rightarrow 1$

$1/k_F a \rightarrow 0^+$ (BEC Limit): repulsive Bose-Einstein condensate of dimers, $\xi \rightarrow 0$

Asymptotic expansion



Lee-Huang-Yang expansion: $E = \frac{N}{2} E_b + N \frac{\pi \hbar^2 a_{\text{mm}}}{2m} n \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a_{\text{mm}}^3} + \dots \right]$

Lee-Yang expansion:

$$E = \frac{3NE_F}{5} \left[1 + \frac{10}{9\pi} k_F a + \frac{4(11 - 2\ln 2)}{21\pi^2} (k_F a)^2 + \dots \right]$$

Lee & Yang *Phys. Rev.* **105**, 1119

(1957) (*repulsive fermions*)

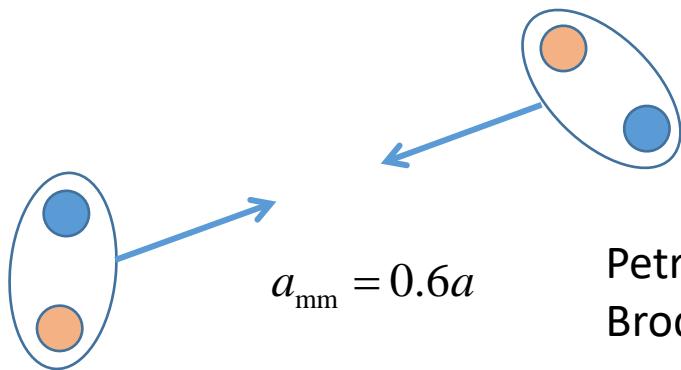
Diener *et al.* *Phys. Rev. A* **77**, 023626

(2008). (*attractive fermions*)

Lee, Huang, Yang, *Phys. Rev.* **106**, 1135 (1957)

X. Leyronas & R. Combescot *PRL* **99**, 170402 (2007) (*Composite bosons*)

Lee-Huang-Yang corrections and few-body physics



Petrov *et al.* PRL **93**, 090404 (2004) (Schrödinger equation)
Brodsky *et al.* PRA **73**, 032724 (2006) (Diagrammatic)

How far can one describe the dimers as point-like bosons?

Higher order expansion (Wu 1957, Braaten et al. PRL 2002)

$$E = \frac{N}{2} E_b + N \frac{\pi \hbar^2 a_{\text{mm}}}{2m} n \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a_{\text{mm}}^3} + \frac{8}{3} (4\pi - 3\sqrt{3}) n_{\text{mm}}^3 \ln(B n a_{\text{mm}}^3) + \dots \right]$$

Non-universal term:
depends on the internal
structure of the boson.

The Unitary Fermi gas ($|a|=\infty$)

$$E = \frac{3NE_F}{5} \xi(0) \sim \text{Ideal Fermi gas. } \xi(0) = \text{« Bertsch parameter »}$$

- BCS mean-field theory: $\xi(0)<0.6$
- DFT (McNeil Forbes *et al.*, PRL 2011): $\xi(0)<0.38(1)$
- Current experimental estimate (ENS-MIT): 0.37(1)

At the surface of a neutron star:

$$k_F a \sim 10; a / r_0 \sim 10$$

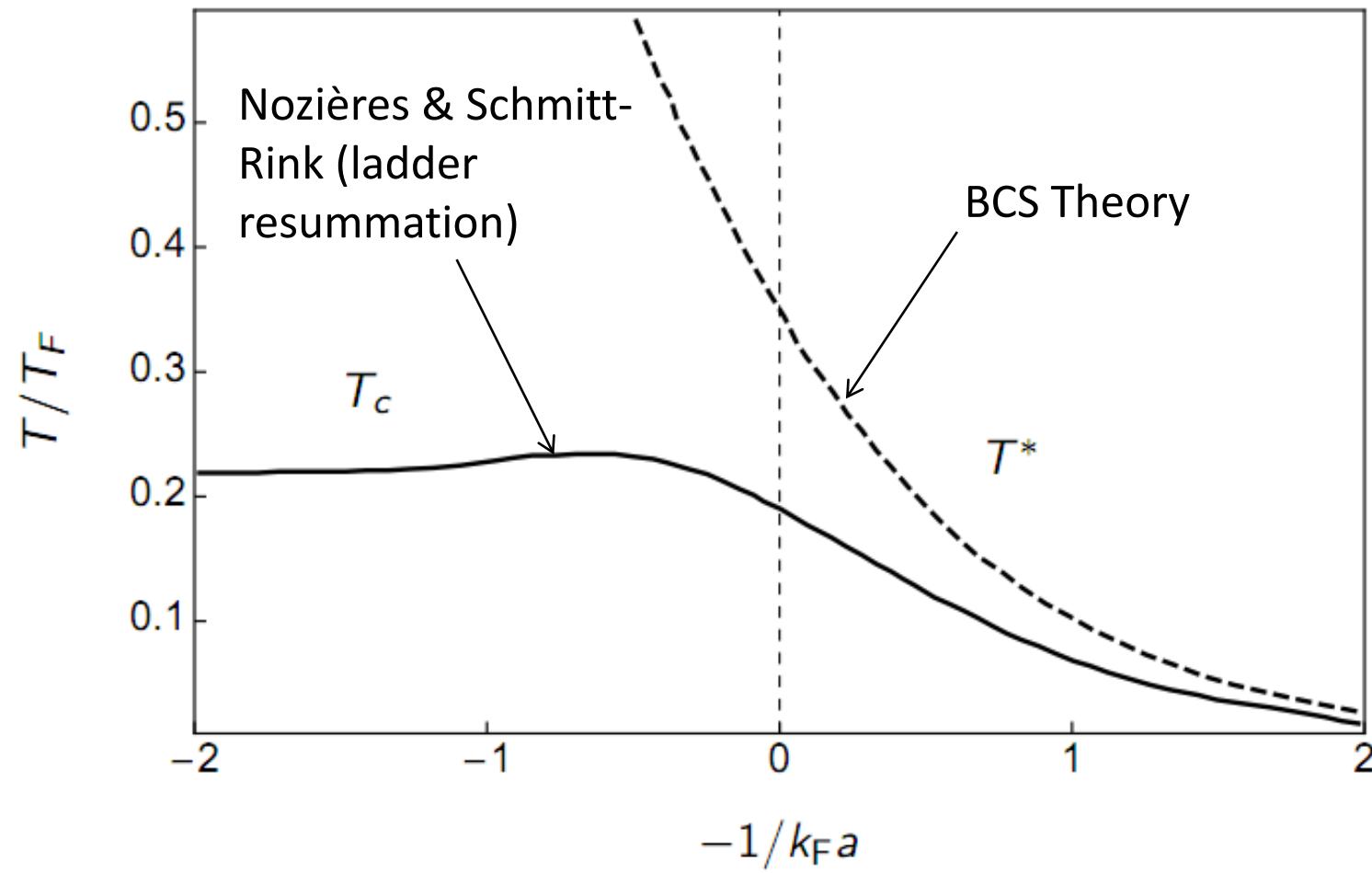


Thermodynamics of a strongly correlated Fermi gas

Finite-temperature equation of state

Critical temperature of the Fermi gas

Shortcomings of the BCS theory:



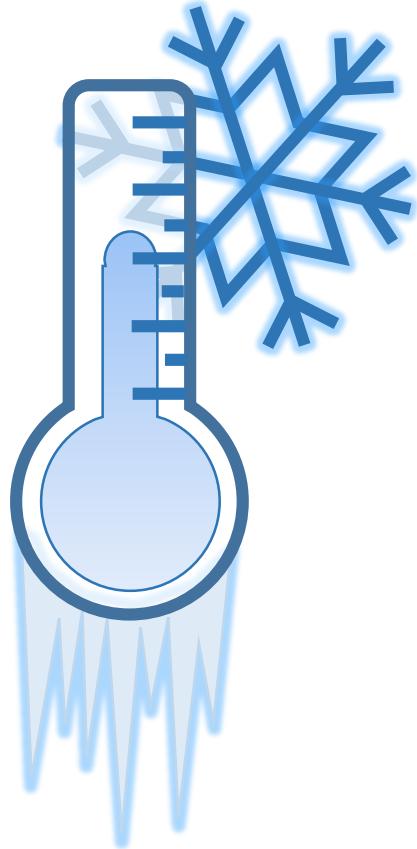
Thermometry of strongly correlated gases?

For a weakly interacting system (~ideal gas): measure the wings of the momentum distribution by time of flight ($\sim e^{-p^2/2mk_B T}$).

For a strongly interacting system: what is the dynamics of the expansion? Even if one « kills » interactions during ToF, what is the relationship between momentum distribution and temperature \Rightarrow heavily model-dependent thermometry.

Three experimental solutions

- Duke: ***entropy measurement*** to determine the microcanonical equation of state
- ENS: use an ***auxiliary thermometer*** (weakly interacting Bose gas mixed to the fermions)
- MIT: measure density (Abel transform) and express $T=f(\text{pressure, compressibility})$ using thermodynamical identities



Measuring the equation of state of a Fermi gas: global thermodynamics

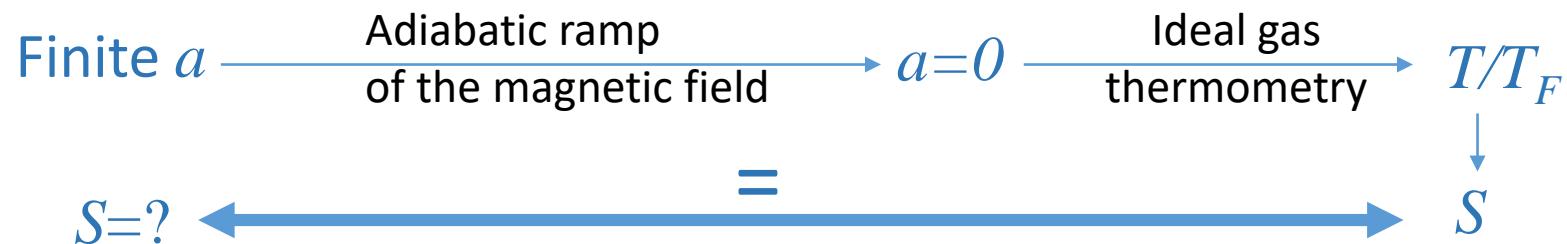
Measure the **microcanonical** equation of state
 $E(S)$

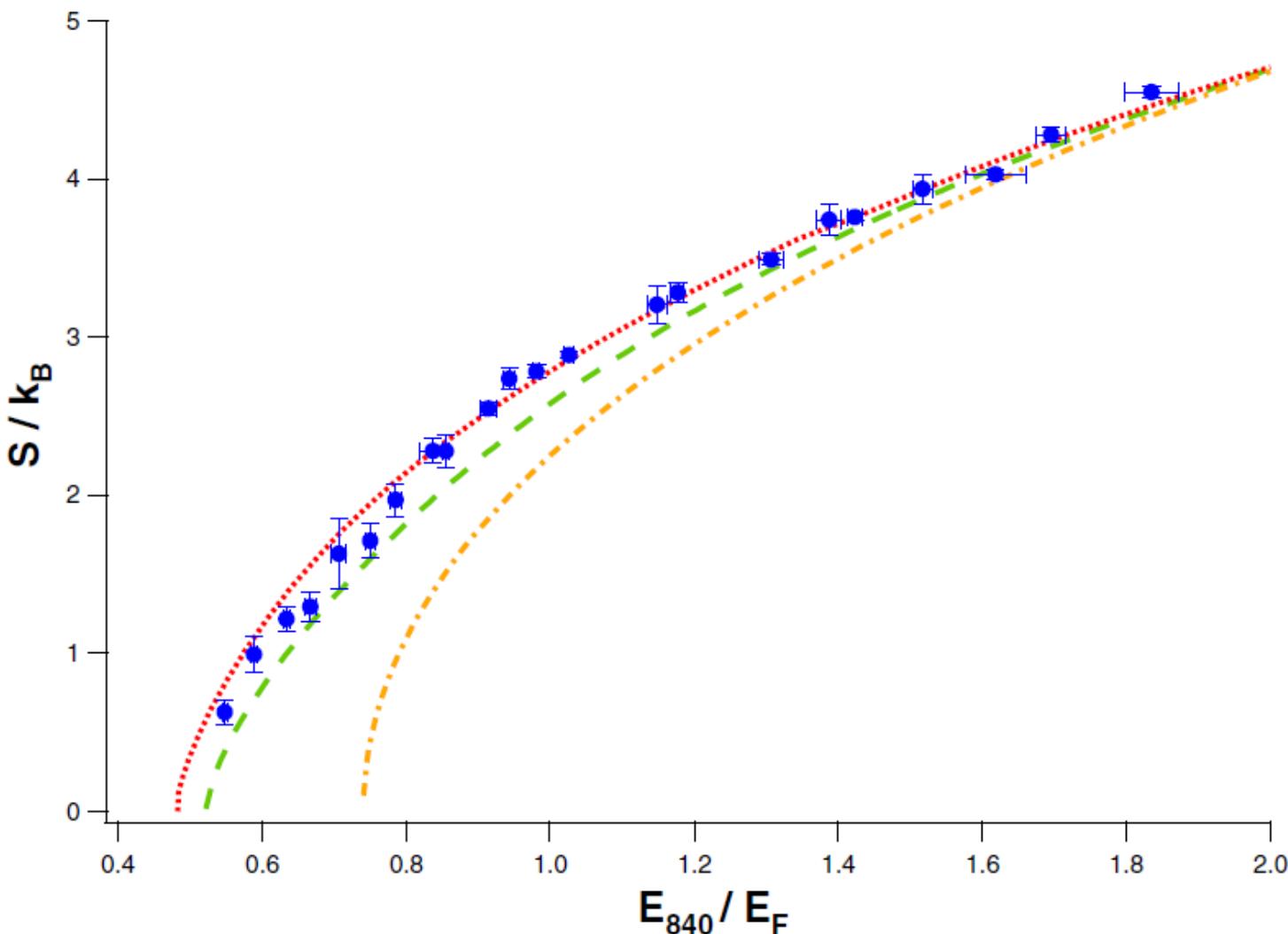
$$E = T + I + V$$

$T + I$ = Released energy, measured by Time of Flight

V = Measured from in-situ imaging

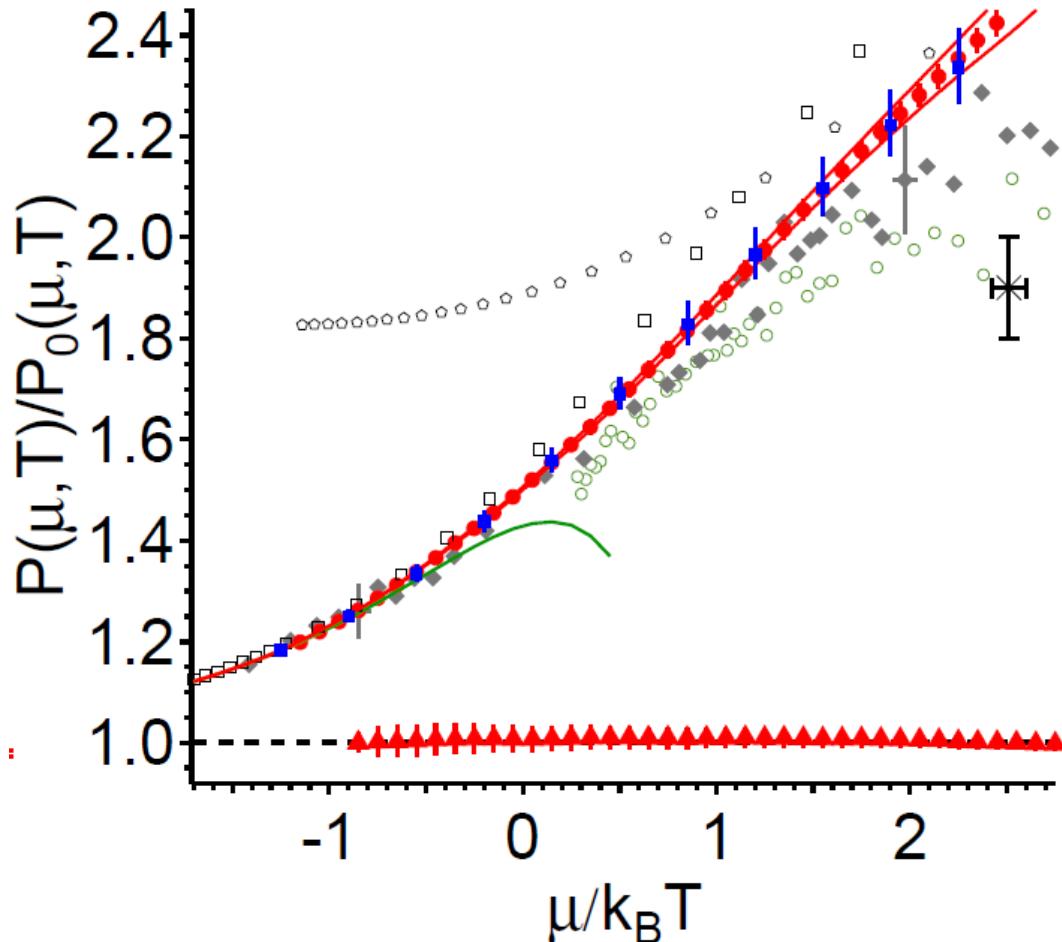
Measurement of the entropy





L. Luo, B. Clancy, J. Joseph, J. Kinast, and J. E. Thomas, Phys. Rev. Lett. **98**, 080402 (2007)

Finite temperature equation of state of the homogeneous unitary Fermi gas.



$P_0(\mu, T)$: equation of state of the ideal Fermi gas.

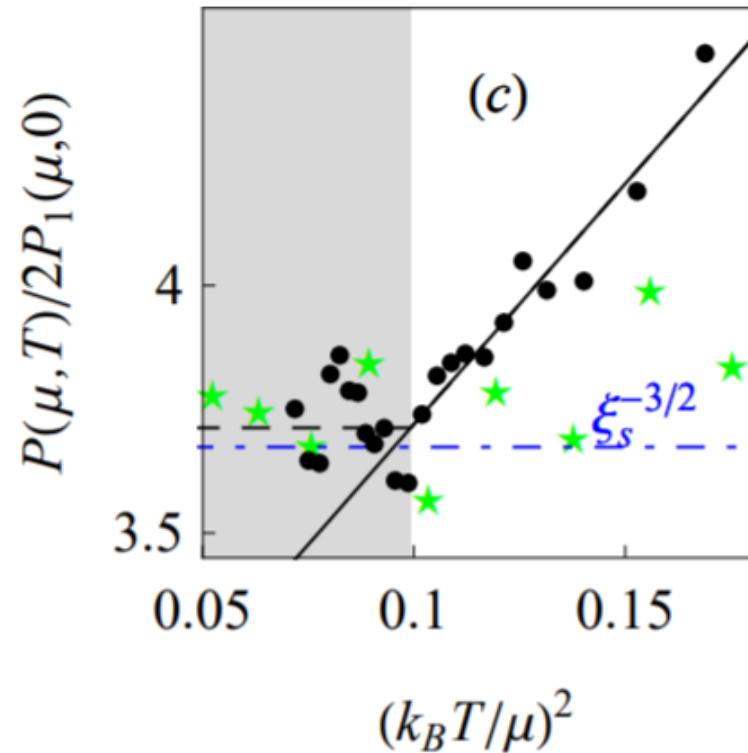
Experiments:

- MIT
- ◆ ENS
- ◇ Tokyo

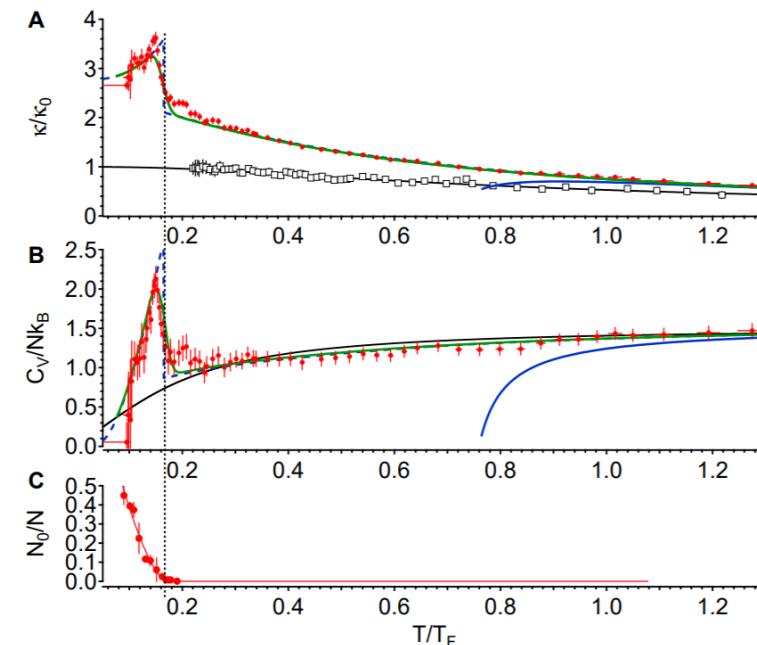
Theory

- Bold Diagrammatic MC
- Auxilliary Field QMC
- First order bold diagram
- 3rd order virial expansion

Revealing the phase transition



ENS: Nascimbène *et al.* Nature (2012)



MIT: Ku *et al.* Science (2012)

$\frac{T_c}{T_F} \approx 0.15$ « high»-T_c superconductivity

High temperature behaviour: virial expansion

Virial expansion for a dilute system: $n\lambda_{th}^3 = 2 \sum_{n=1}^{\infty} kb_k \zeta^k$

$\zeta = e^{\beta\mu}$ =fugacity ($\rightarrow 0$ in the dilute limit)

b_k =dimensionless parameters, related to the k-body problem. Pure numbers at the unitary limit.

	Experiment (ENS)	Theory
b_2	NA	$\frac{1}{\sqrt{2}} - \frac{2}{2^{5/2}}$ (Ho & Mueller, 2004)
b_3	-0.29(2)	-0.291 (Liu <i>et al.</i> , 2009)
b_4	0.064(15)	0.03 (Rossi <i>et al.</i> , 2018) 0.05 (Yan & Blume, 2016)