ÉCOLE DE PHYSIQUE
des HOUCHES

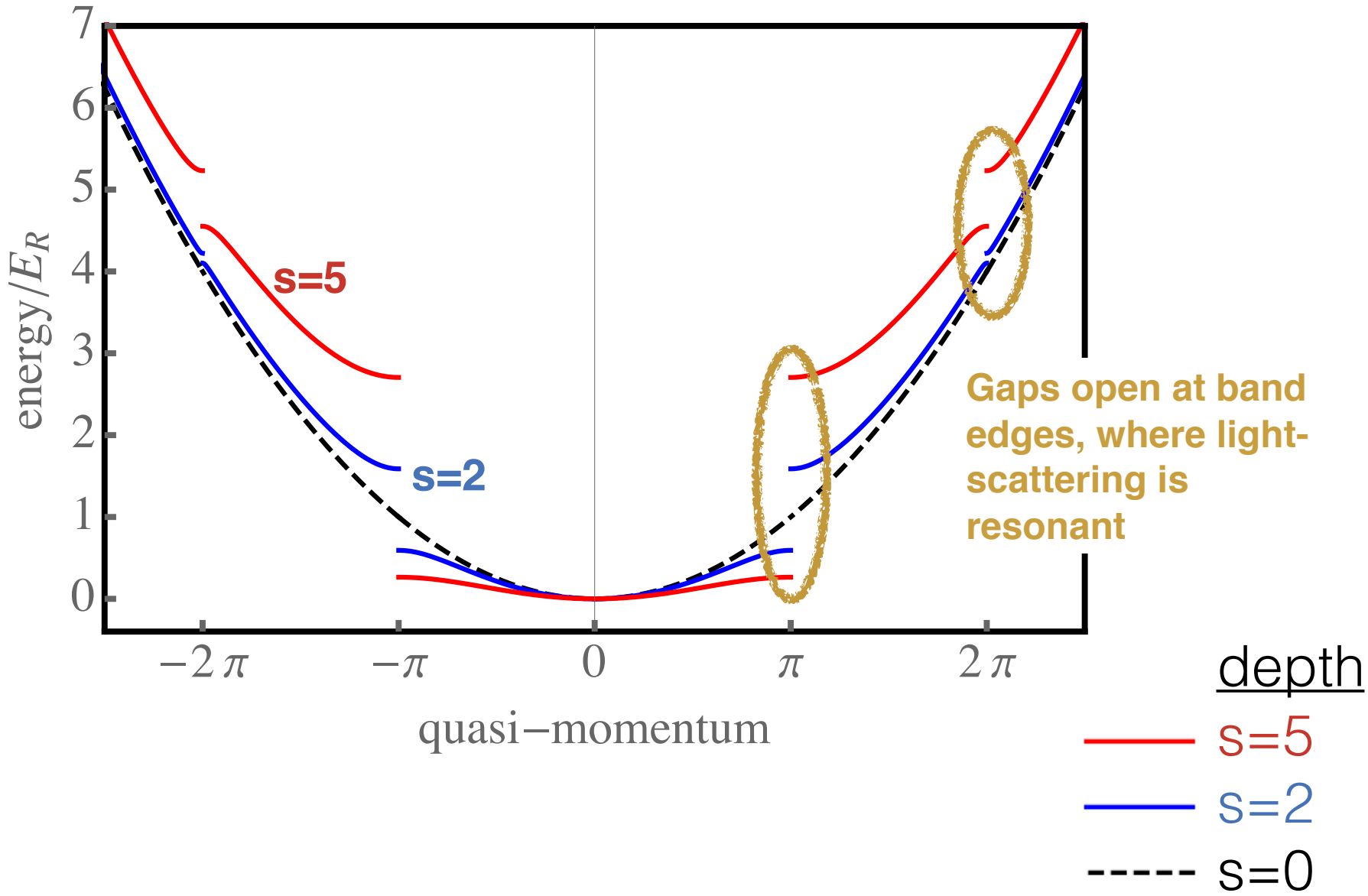
Predoc School on
Ultracold Fermions

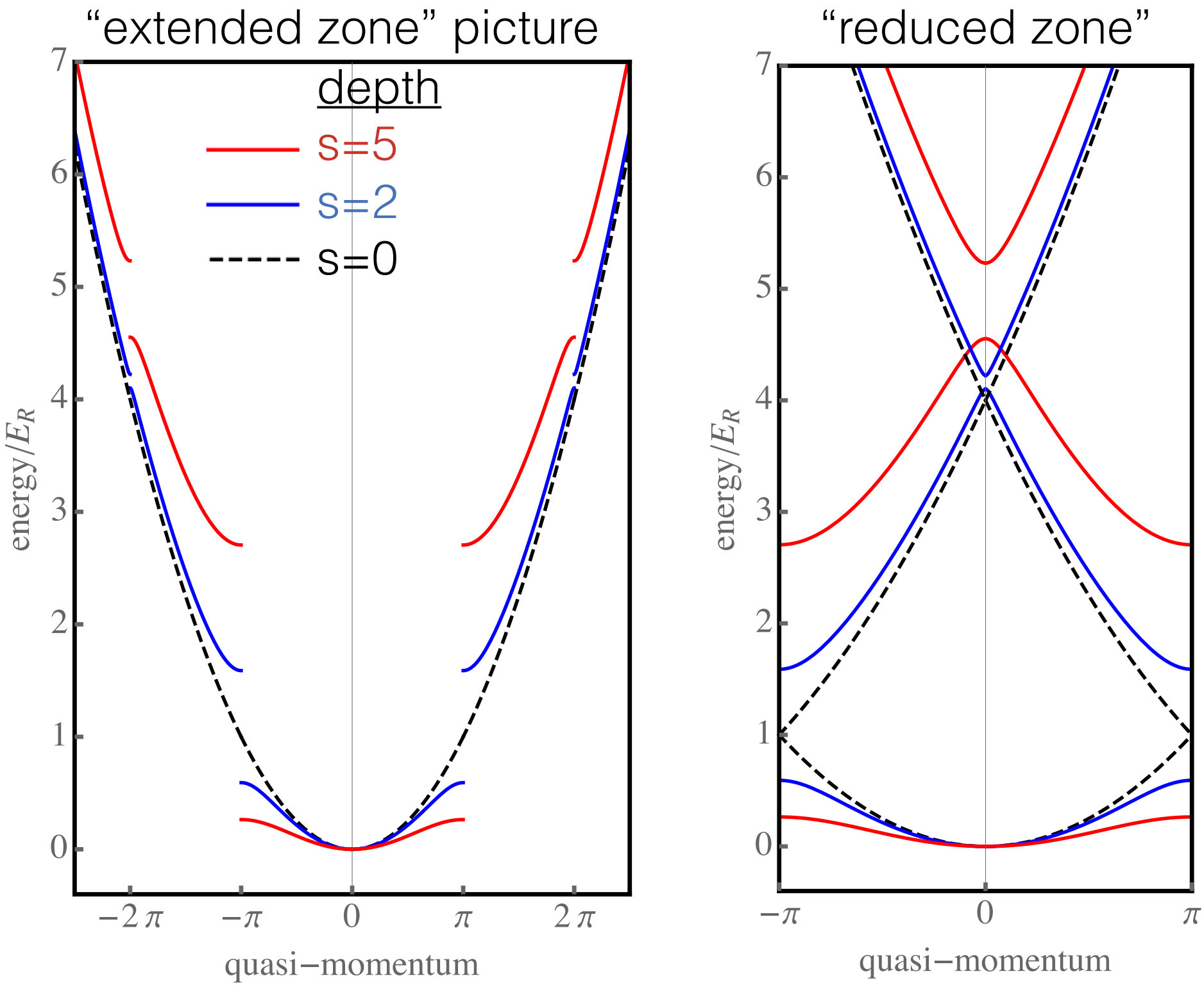
Fermions in Optical Lattices

8-9 October 2018

Joseph H Thywissen
University of Toronto

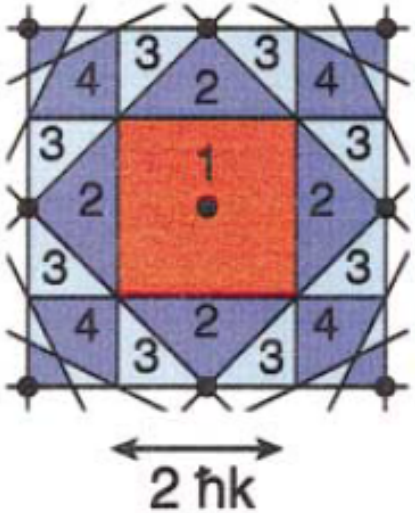
Energies of Bloch states



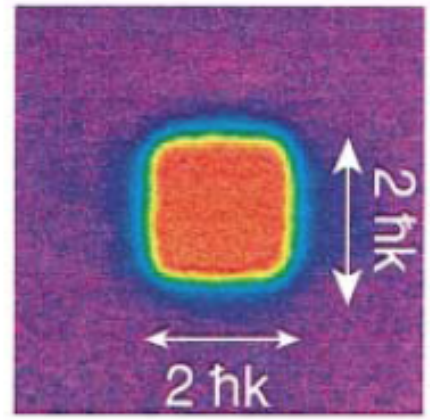

"extended zone" picture
depth
s=5
s=2
s=0
energy/E_R
quasi−momentum
-2π
$-\pi$
0
π
2π
"reduced zone"
energy/E_R
quasi−momentum
$-\pi$
0
π

Seeing band structure: “band mapping”

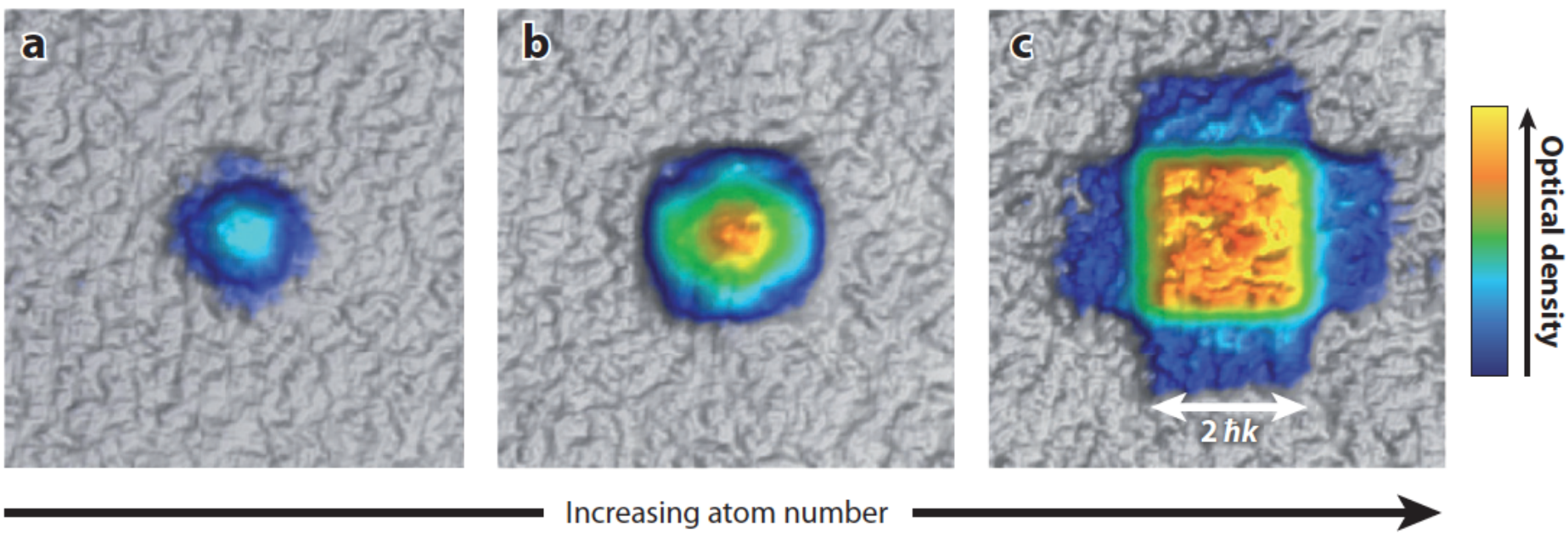
Brillouin zone structure of a 2D square lattice

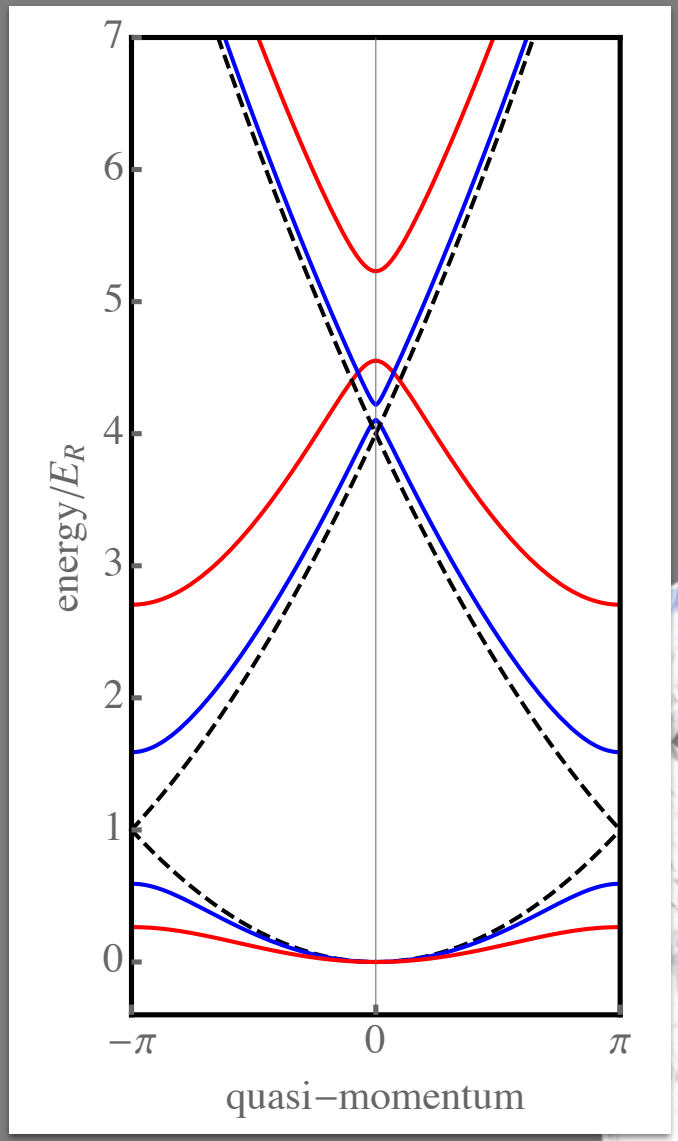


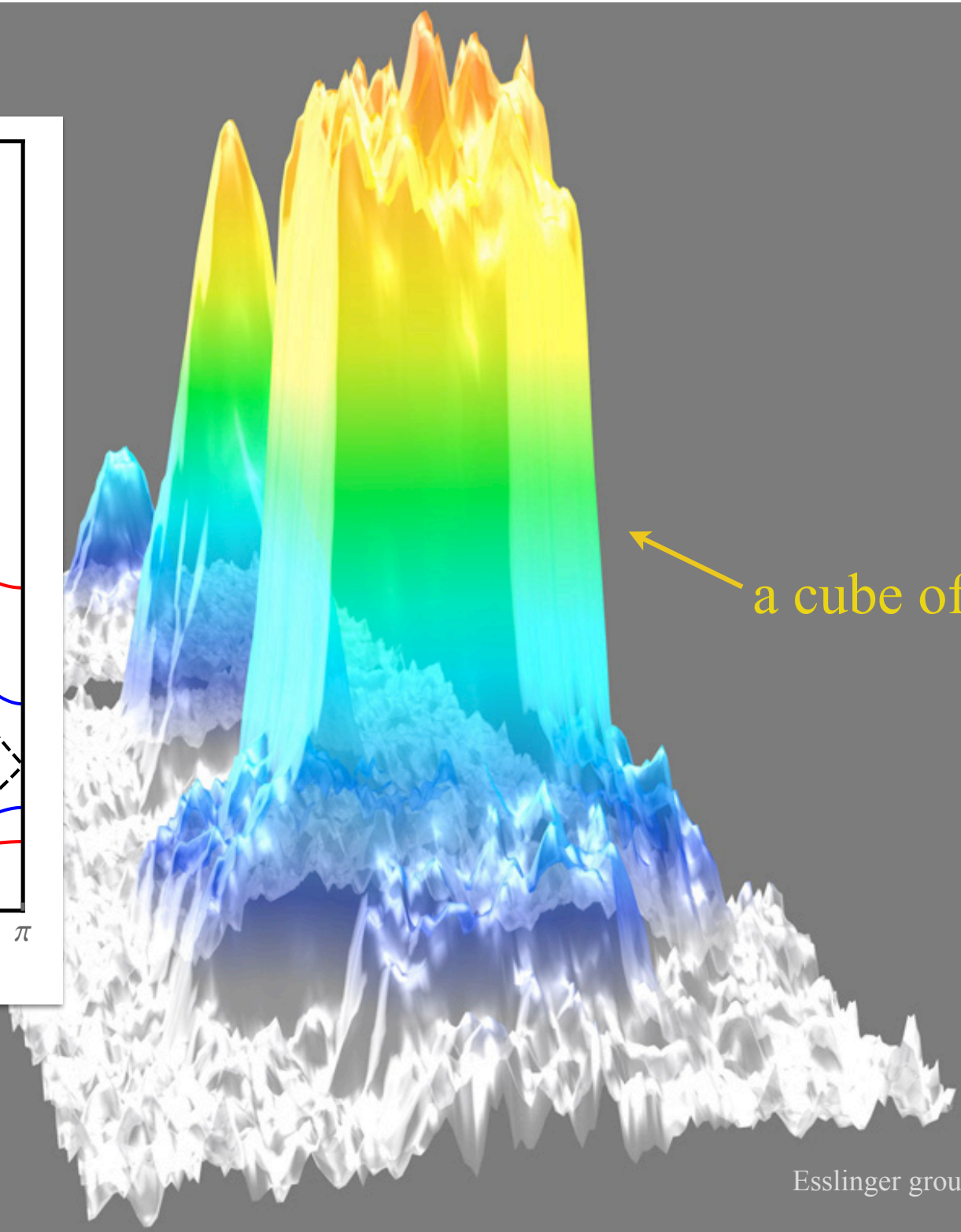
Hansch/Esslinger (2001)



Esslinger ARCMP (2010)

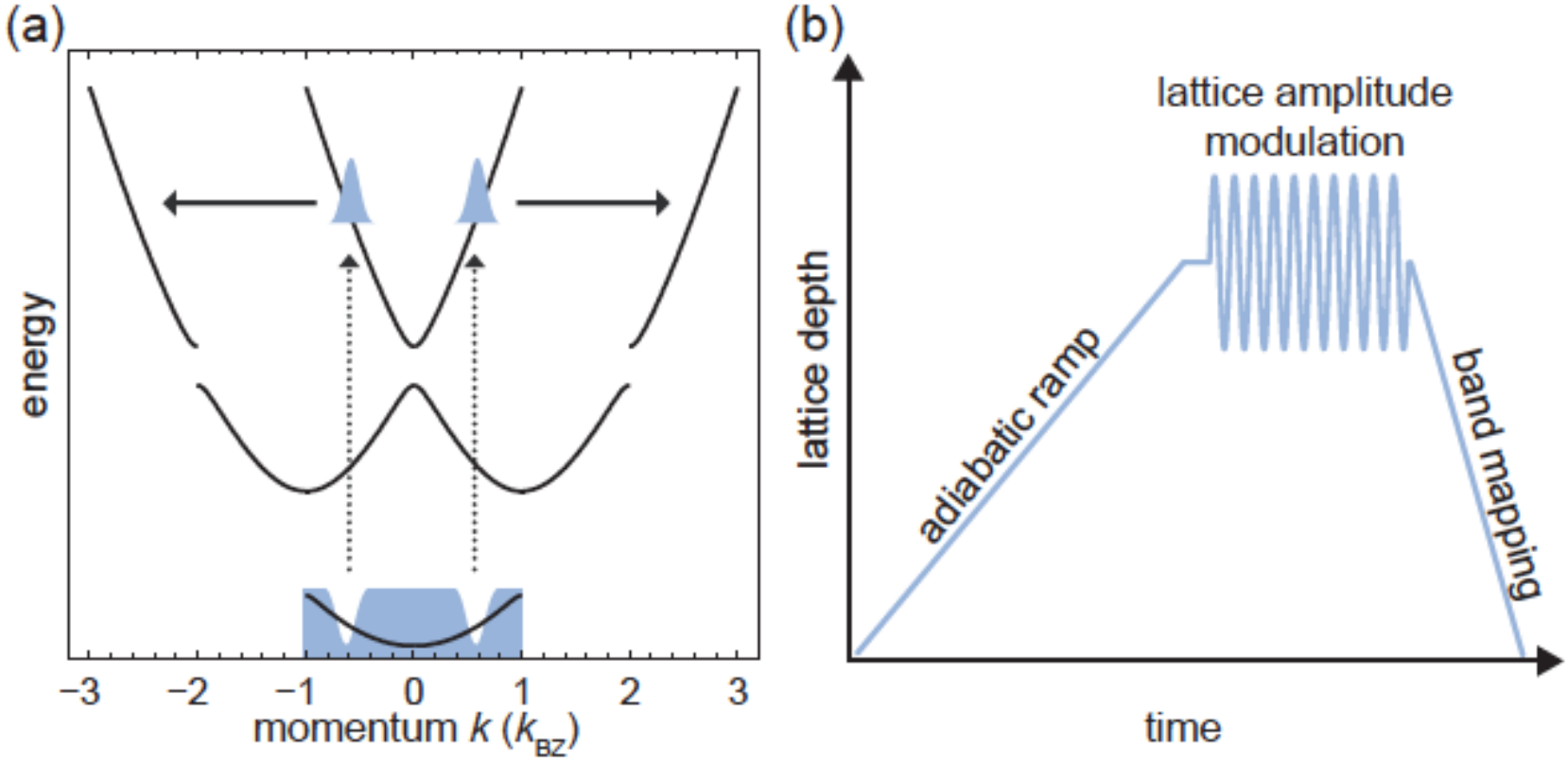


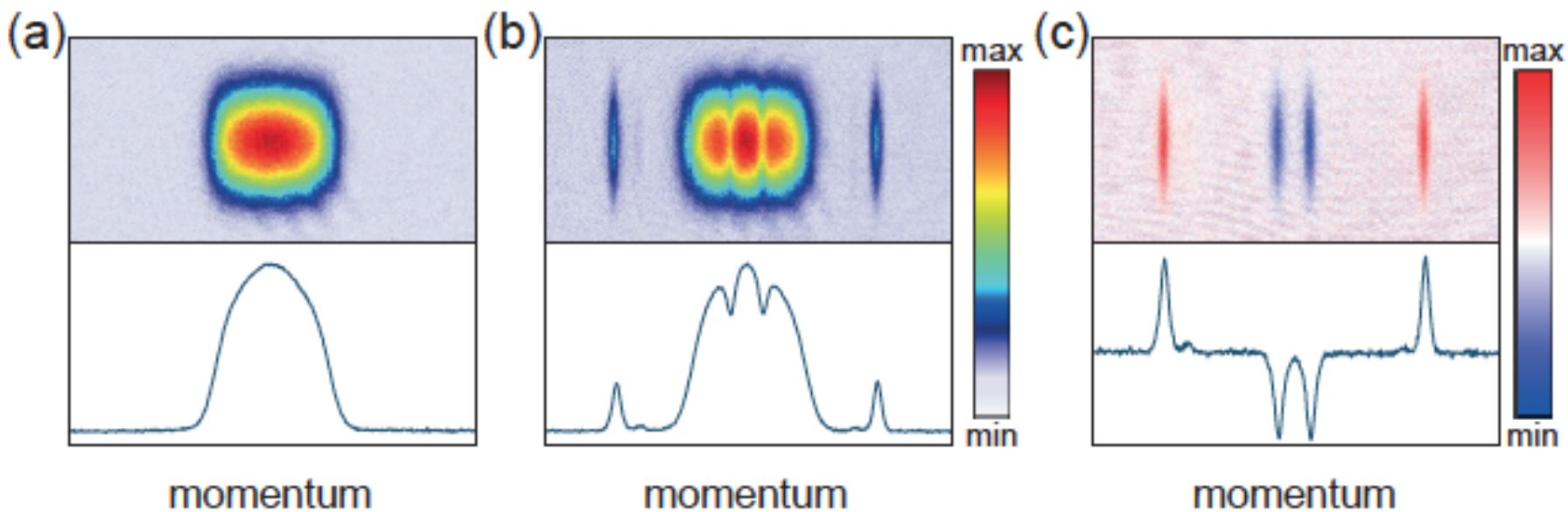




Esslinger group, ETH Zurich

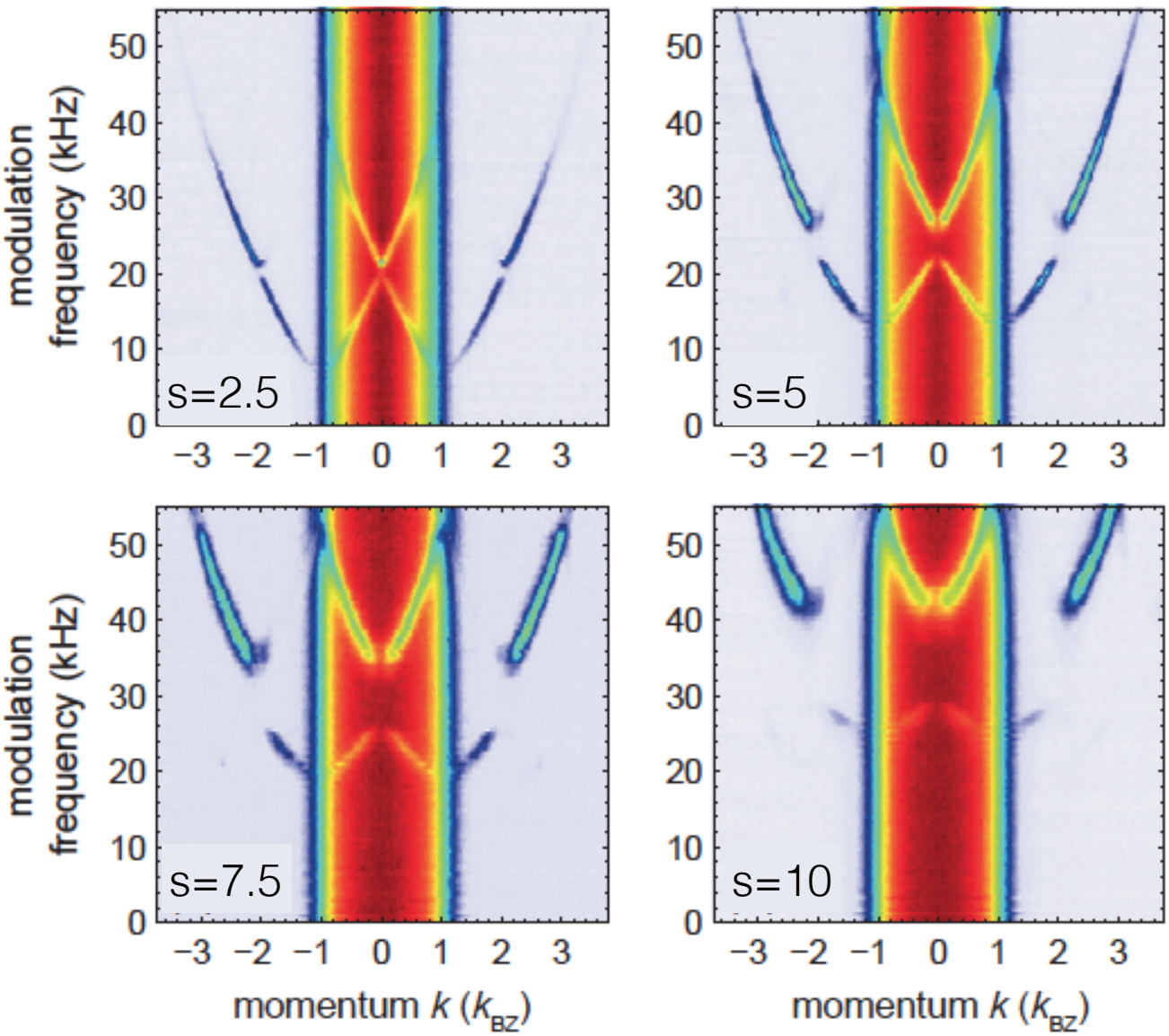
Measuring band structure



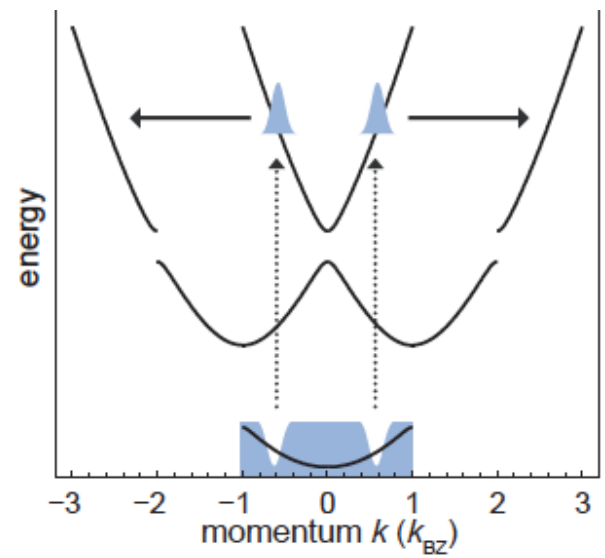


J. Heinze, Ph.D. thesis (Hamburg 2014)

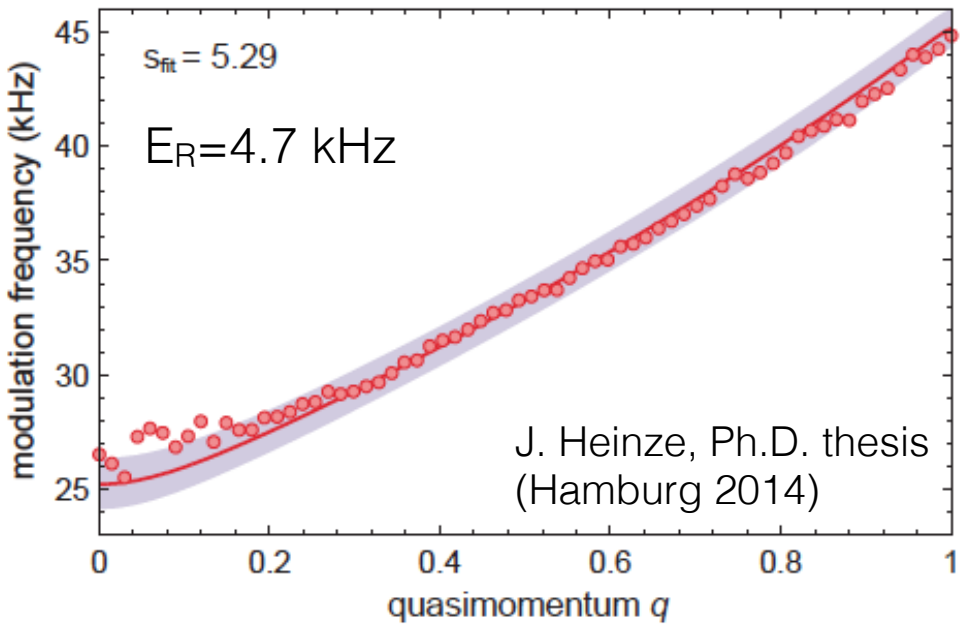
Measuring band structure

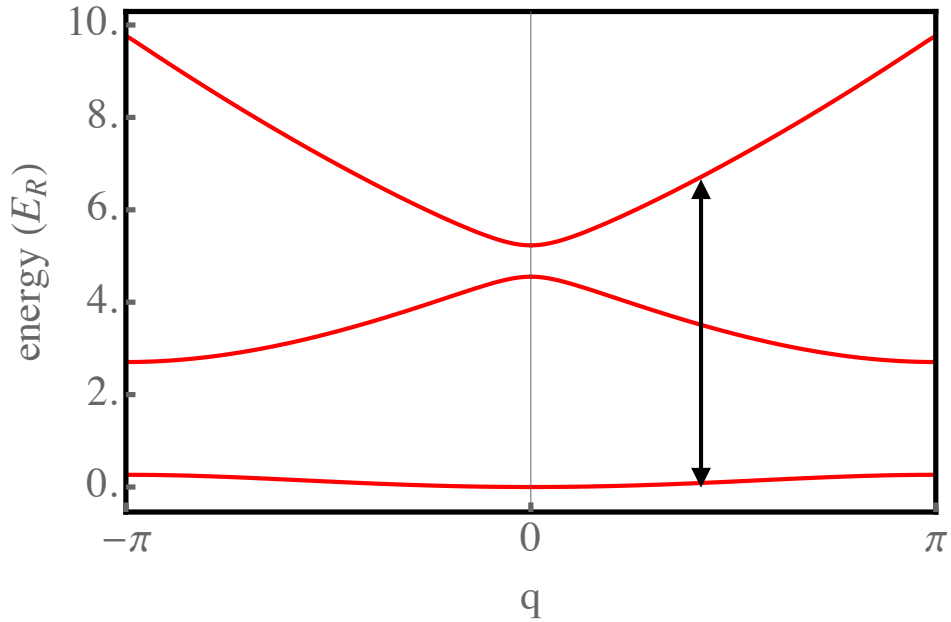


J. Heinze, Ph.D. thesis (Hamburg 2014)



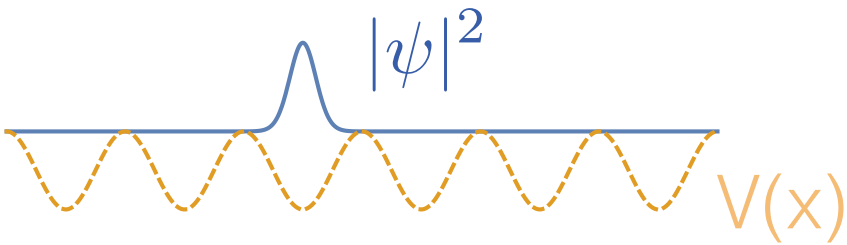
Measuring band structure



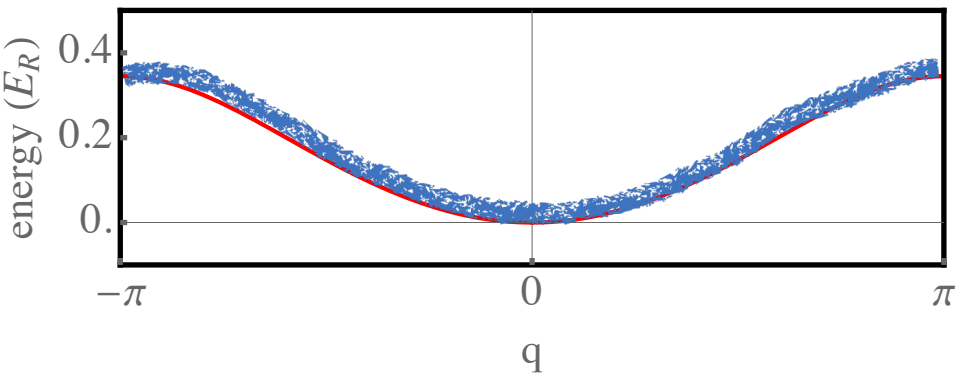


[PS — see Gabriel Chatelain's poster on Direct cooling with amplitude modulation!]

Wannier states

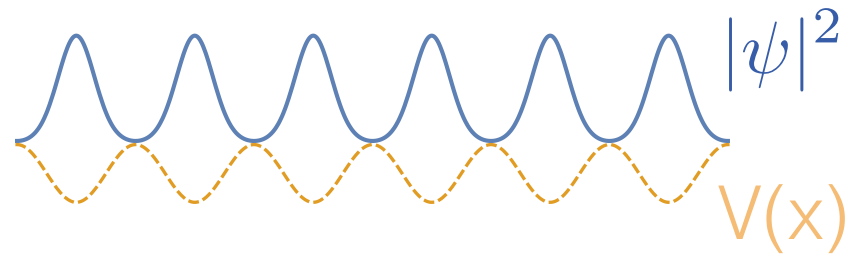


well defined **x**

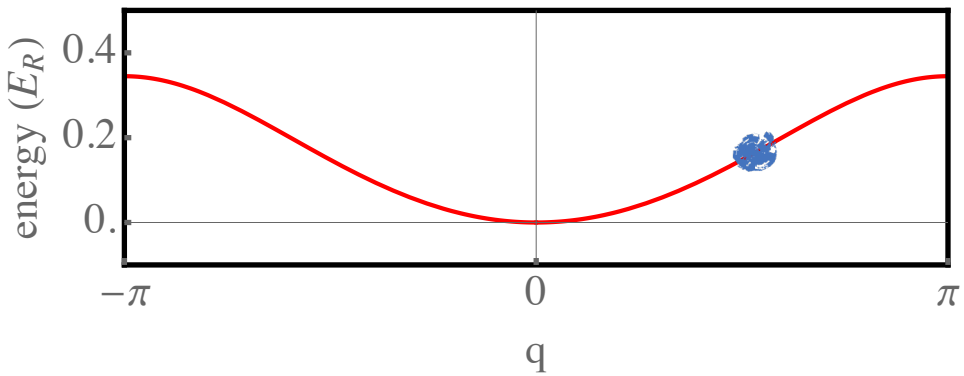


sum over all q

Bloch states



well defined **q**

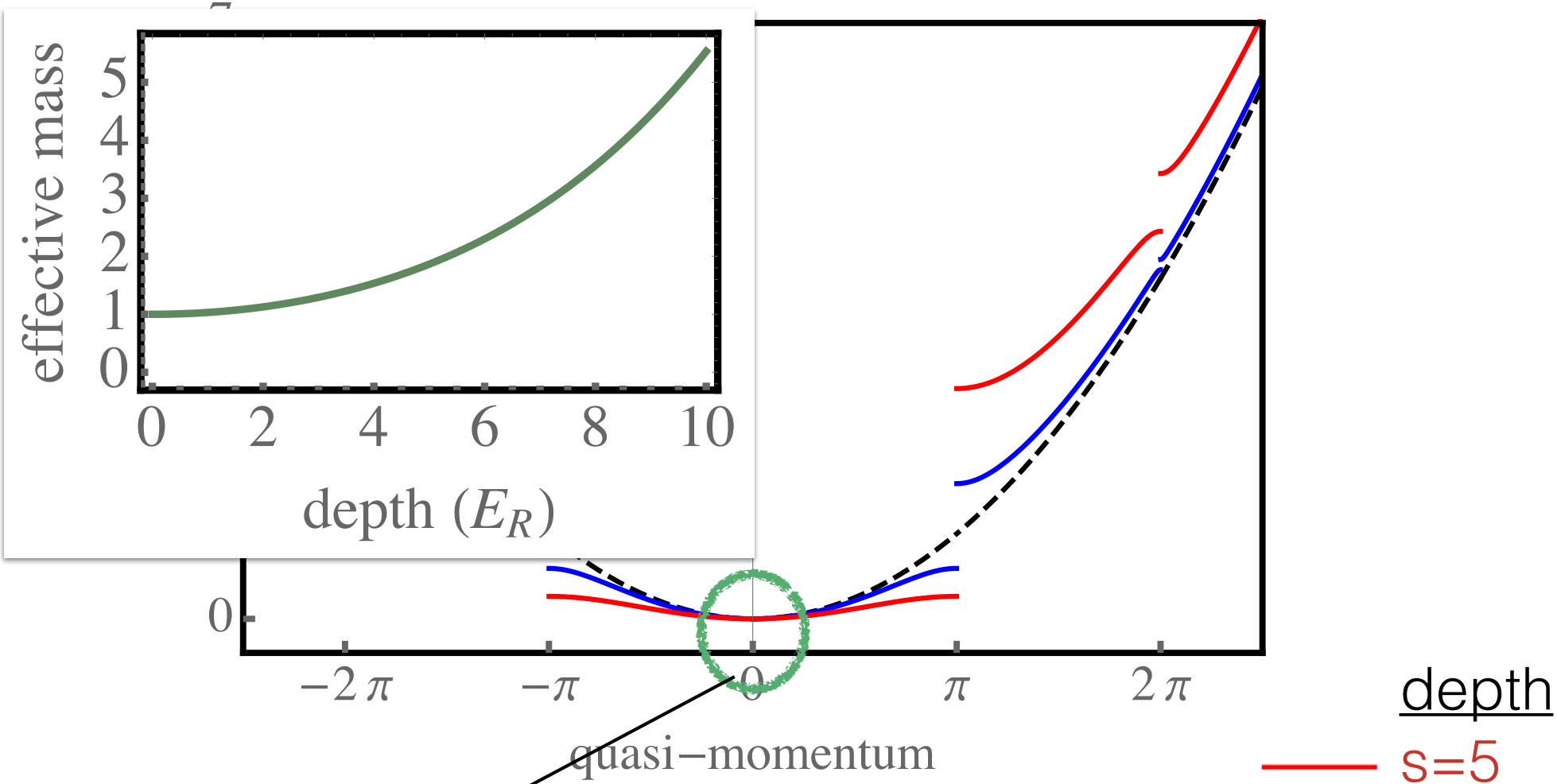


Discrete Fourier relations between creation operators:

$$\hat{c}_\ell^\dagger = \frac{1}{\sqrt{M}} \sum_n^M e^{-iq_n \cdot x_\ell / a_L} \hat{c}_{q_n}^\dagger$$

$$\hat{c}_{q_n}^\dagger = \frac{1}{\sqrt{M}} \sum_\ell^M e^{iq_n \cdot x_\ell / a_L} \hat{c}_\ell^\dagger$$

Effective mass

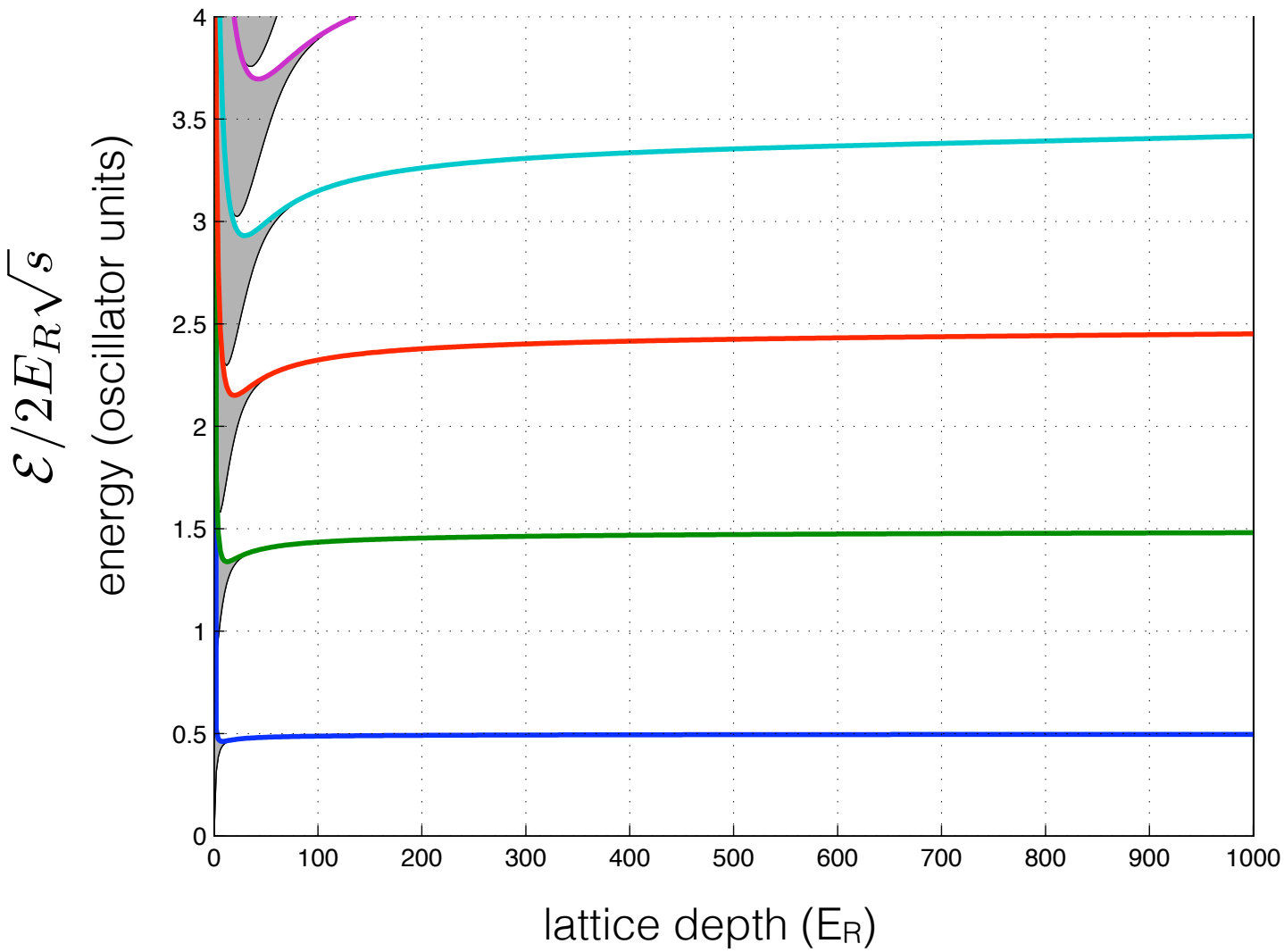


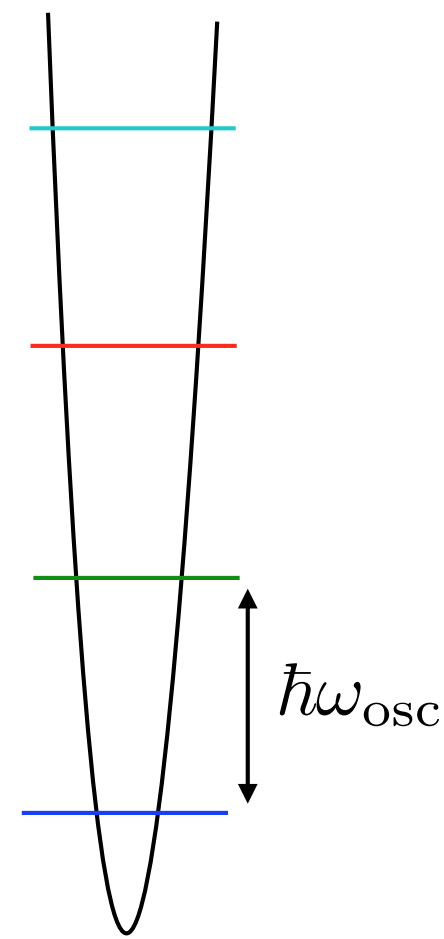
Curvature reduced at q=0, showing a larger effective mass m*.

Deep lattice limit

Isolated wells, zero tunnelling

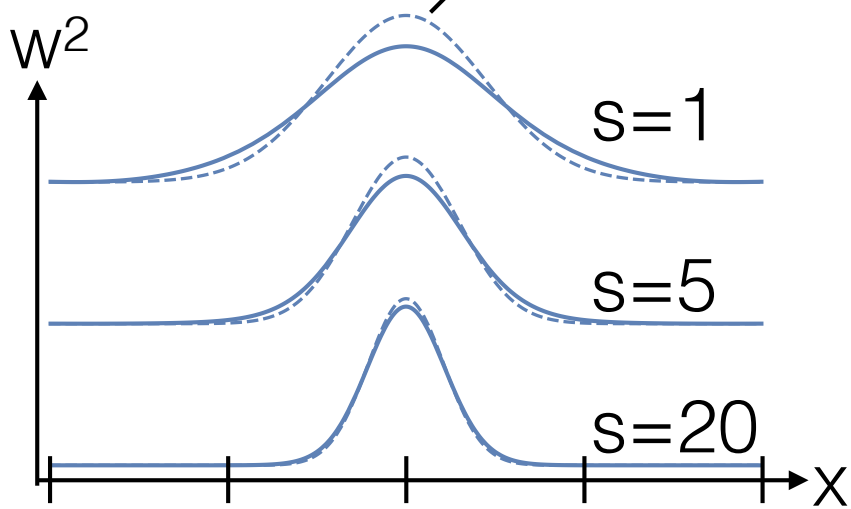
$$\hbar\omega_{\mathrm{osc}} = 2\sqrt{s}E_R$$





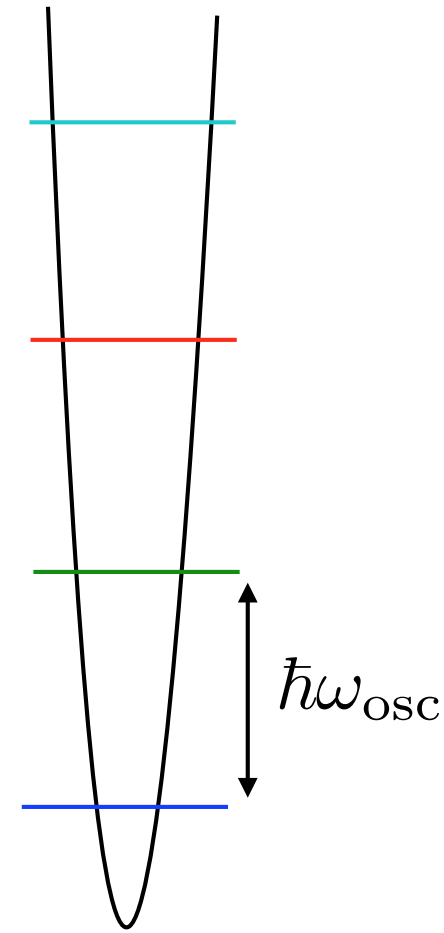
Wannier states approach harmonic oscillator states for a deep lattice:

$$w_0(x) \sim \exp\left[-\frac{x^2}{2\sigma_{\mathrm{HO}}^2}\right]$$



$$\sigma_{\mathrm{HO}} = s^{-1/4} a_L/\pi$$

$$\hbar\omega_{\mathrm{osc}} = 2\sqrt{s}E_R$$



Summary, Topic 2

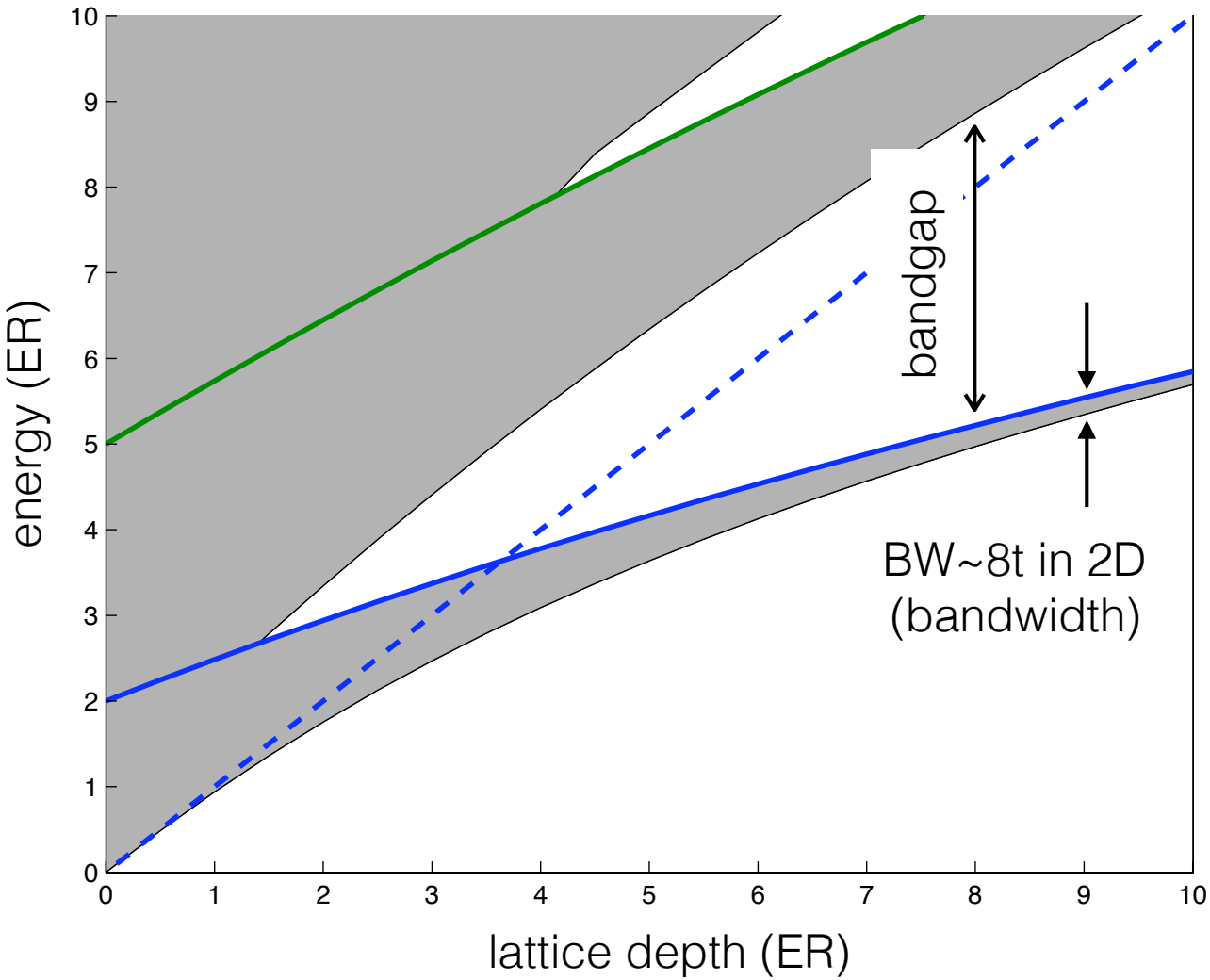
- *Matter waves in a crystal of light*: single-particle eigenstates are delocalised *Bloch states*
- Band gaps show strength of **Bragg scattering** from lattice beams
- **Quasi-momentum** is not atomic momentum: atoms in a lattice are coherently dressed with photons
- The Fourier components of a band are the tunnelling coefficients
- Observations: Band mapping; Fermi surface in the band gap (a cube of gas); inter-band excitation

ÉCOLE DE PHYSIQUE
des HOUCHES

Fermions in Optical Lattices

- Introduction
- Fermions, statistics, & exchange
- Matter waves in crystals of light
- The Hubbard model
- Transport

Isolation of a single band

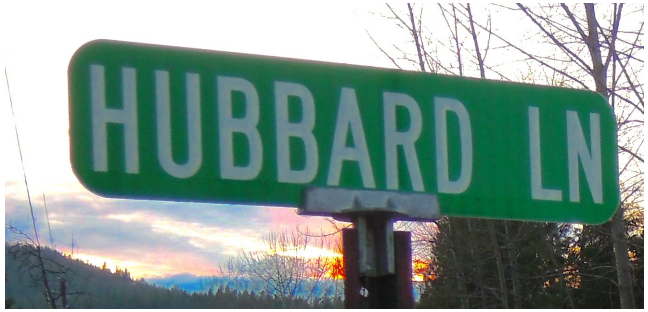


<u>for s=8:</u>

$t=0.03E_R=6.6nK=140Hz$

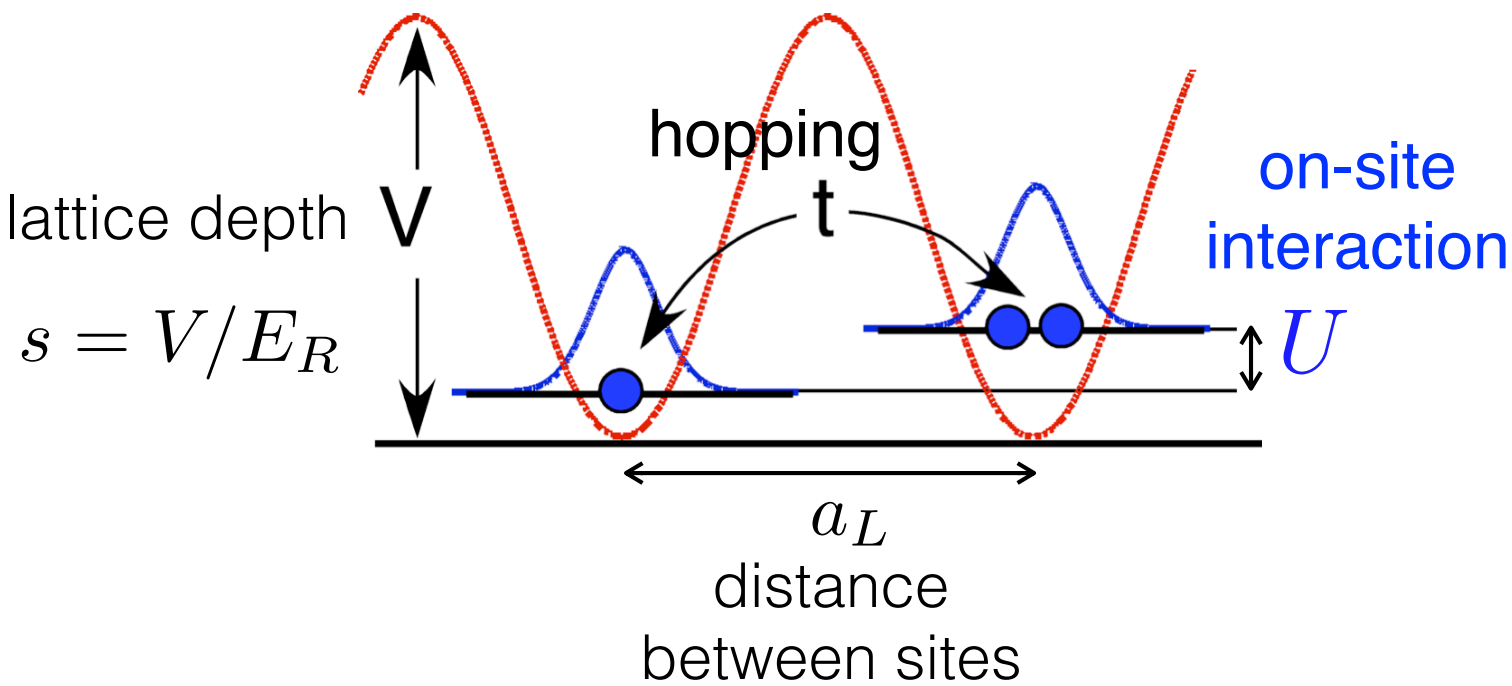
$bandgap=3.6E_R=0.8\mu K=16kHz$

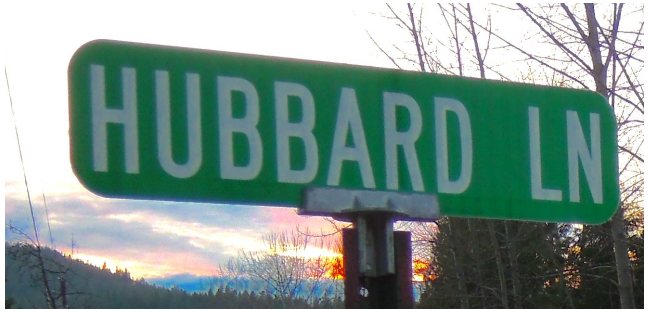
The Hubbard Model



The Hubbard Model

$$\hat{H}_{\mathrm{HM}} = -t \sum_{\langle j,\ell \rangle,\sigma} \hat{c}^{\dagger}_{j\sigma} \hat{c}_{\ell\sigma} + U \sum_{\ell} \hat{n}_{\ell\uparrow} \hat{n}_{\ell\downarrow}$$





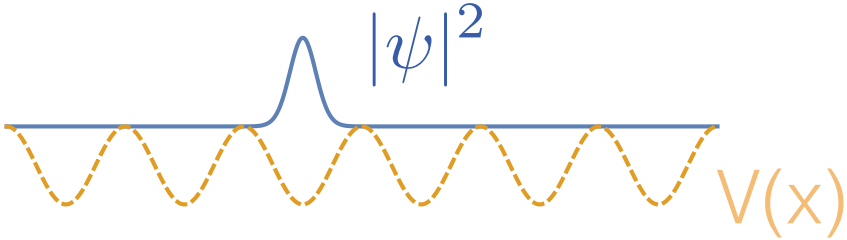
Local interactions

$$\hat{H}_{\mathrm{HM}} = -t \sum_{\langle j,\ell \rangle,\sigma} \hat{c}^{\dagger}_{j\sigma} \hat{c}_{\ell\sigma} + U \sum_{\ell} \hat{n}_{\ell\uparrow} \hat{n}_{\ell\downarrow}$$

$$\hat{H}_{\mathrm{HM}} = \sum_{q,\sigma} \mathcal{E}(q) \hat{n}_{q\sigma} + \boxed{?}$$

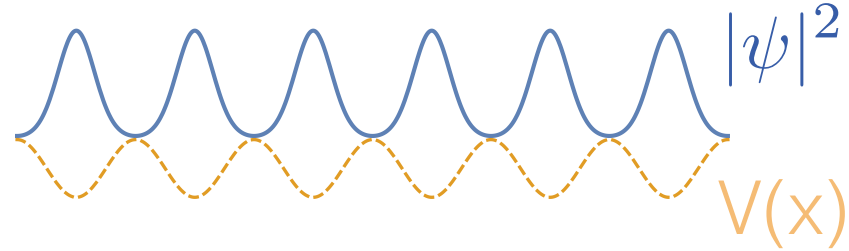
Bloch waves

Wannier states



well defined **x**

Bloch states

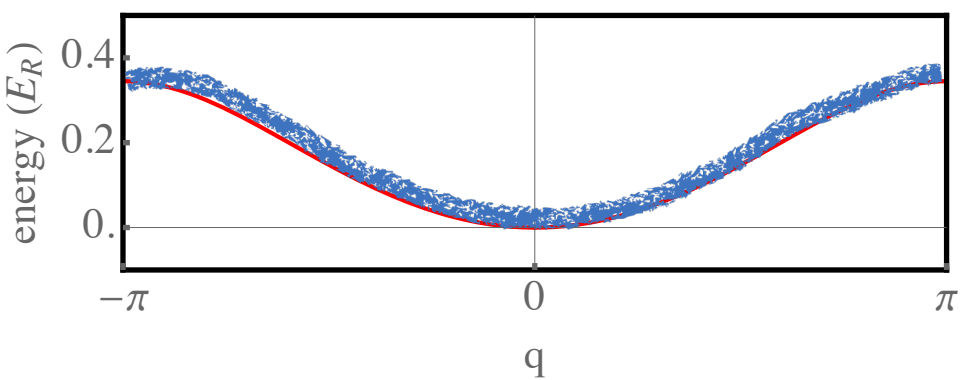


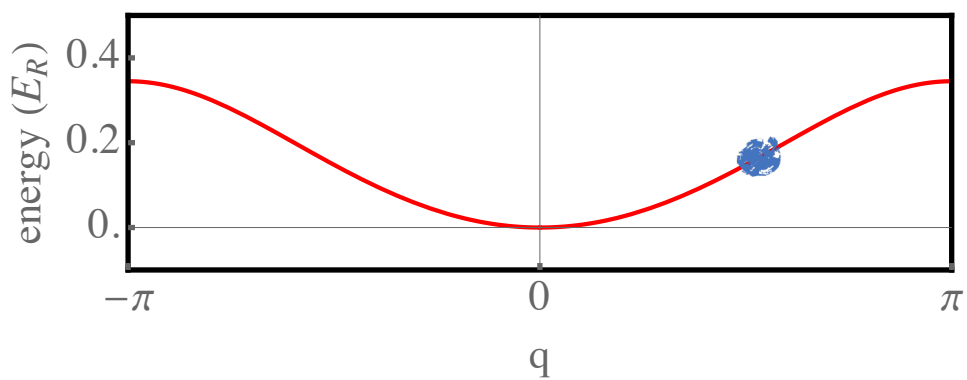
well defined **q**

Discrete Fourier relations between creation operators:

$$\hat{c}^\dagger_\ell = \frac{1}{\sqrt{M}} \sum_n^M e^{-iq_n \cdot x_\ell / a_L} \hat{c}^\dagger_{q_n}$$

$$\hat{c}^\dagger_{q_n} = \frac{1}{\sqrt{M}} \sum_\ell^M e^{iq_n \cdot x_\ell / a_L} \hat{c}^\dagger_\ell$$





sum over all q

Interaction term in Bloch basis

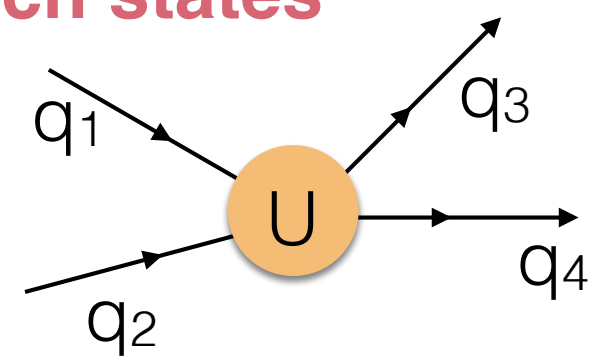
$$\hat{H}_U = U \sum_{\ell} \hat{n}_{\ell\uparrow}\hat{n}_{\ell\downarrow}$$ *on-site interaction*

$$\vdots$$

$$\hat{H}_U = \frac{U}{M} \sum_{q_1,q_2,q_3} \hat{c}^\dagger_{q_4\uparrow}\hat{c}^\dagger_{q_3\downarrow}\hat{c}_{q_2\downarrow}\hat{c}_{q_1\uparrow}$$

where $q_4 = q_1 + q_2 - q_3 \bmod 2\pi$

scattering between Bloch states



Scattering preserves quasi-momentum, not true momentum, in a lattice!

Hubbard model from two perspectives

$$\hat{H}_{\mathrm{HM}} = \hat{H}_0 + \hat{H}_{\mathrm{U}}$$

Localized basis:

$\hat{H}_0 =$ **hopping to new site**

$$-t \sum_{\langle j,\ell \rangle,\sigma} \hat{c}^\dagger_{j\sigma} \hat{c}_{\ell\sigma}$$

$\hat{H}_{\mathrm{U}} =$ **local interactions**

$$U \sum_{\ell} \hat{n}_{\ell\uparrow} \hat{n}_{\ell\downarrow}$$

Delocalized basis:

$\hat{H}_0 =$ **Bloch waves**

$$\sum_{q,\sigma} \mathcal{E}(q) \hat{n}_{q\sigma}$$

$\hat{H}_{\mathrm{U}} =$ **atom-atom scattering**

$$\frac{U}{M} \sum_{q_1,q_2,q_3} \hat{c}^\dagger_{q_4\uparrow} \hat{c}^\dagger_{q_3\downarrow} \hat{c}_{q_2\downarrow} \hat{c}_{q_1\uparrow}$$

Hubbard model: observed phases, & quantum gas microscopes

Hubbard model: observed phases, & quantum gas microscopes

Bosonic SF-to-Mott Insulator transition

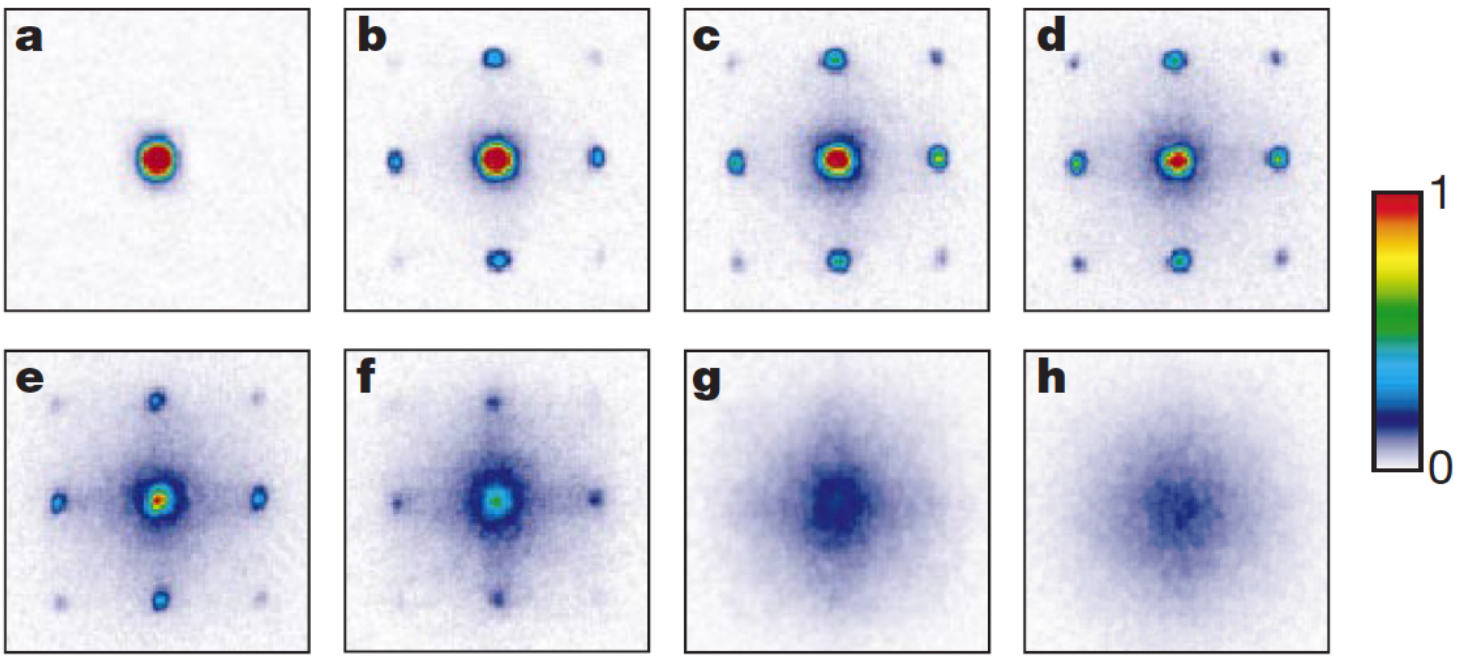
Quantum Gas microscopy

Phases of Fermions in optical lattices

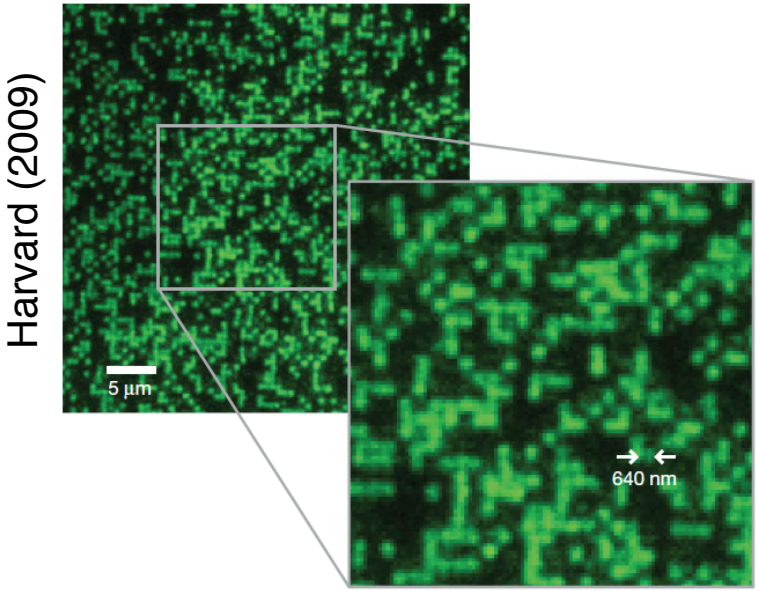
Superfluid -to- Mott Insulator transition

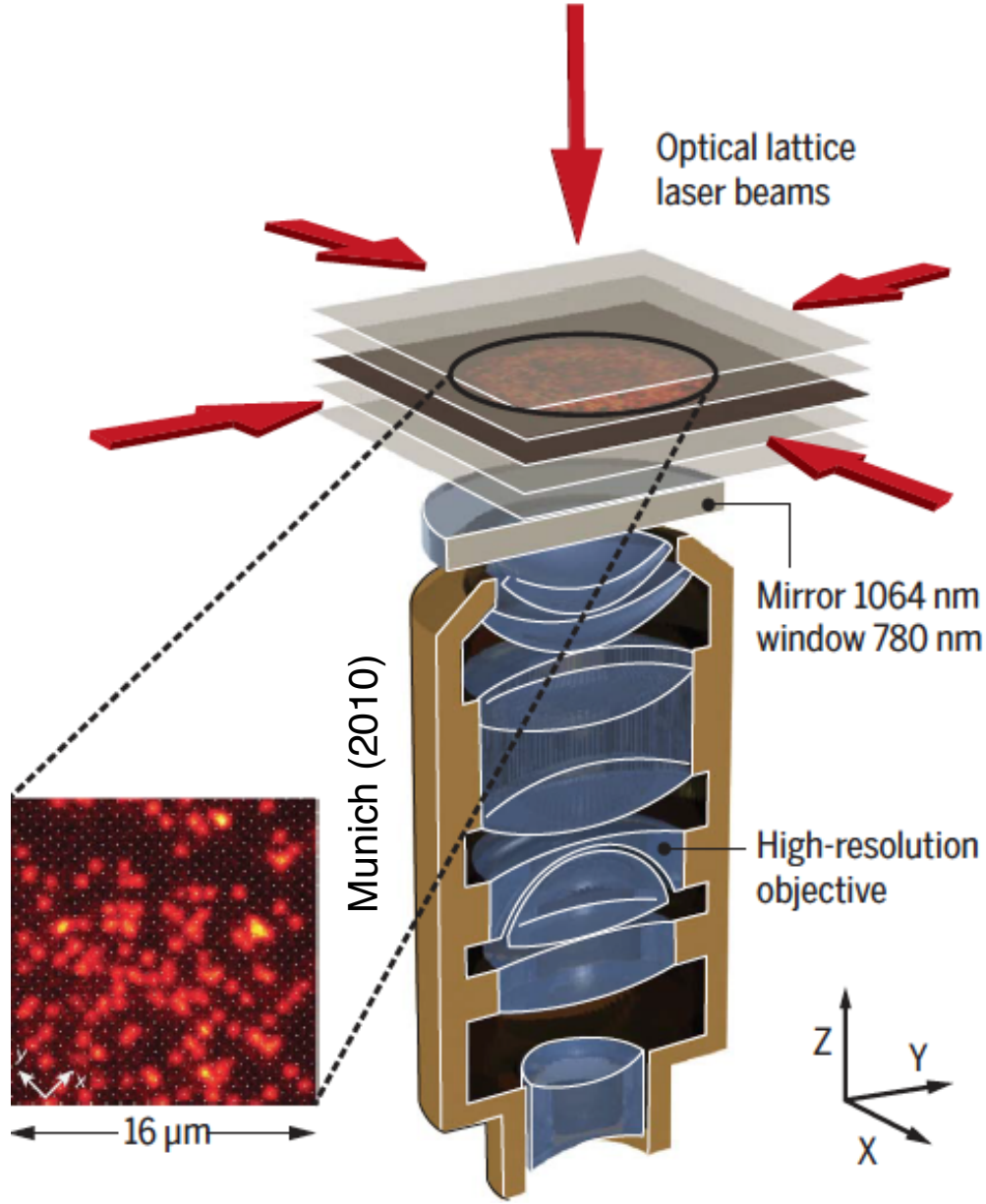
$$|\Psi_{\mathrm{MI}}\rangle_{J=0} \propto \prod_{i=1}^{M} (\hat{a}_i^\dagger)^n |0\rangle \qquad |\Psi_{\mathrm{SF}}\rangle_{U=0} \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger \right)^N |0\rangle$$

Munich (2002)



Bosonic Mott Insulator experiments:
Munich, NIST/JQI, ETHZ, MIT, Austin, …



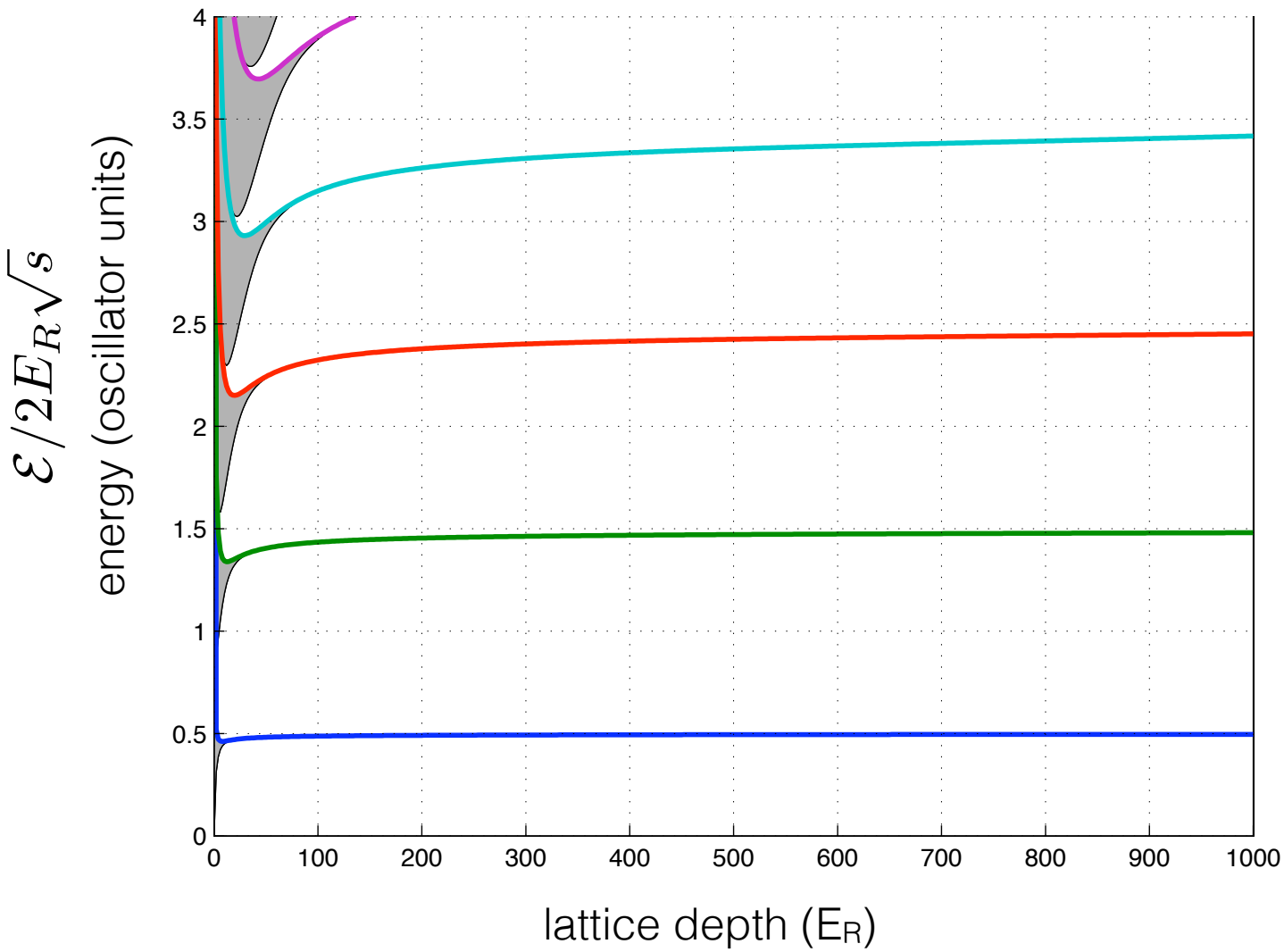


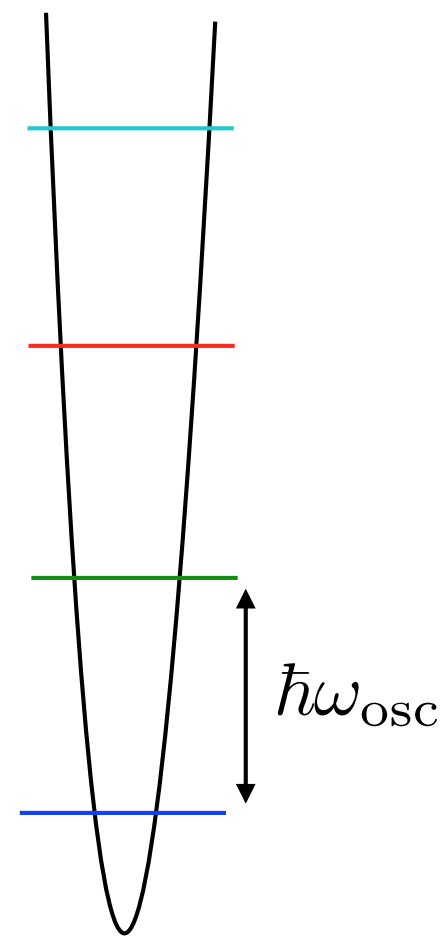
Bosonic quantum gas microscopes:
Harvard, MPQ, Tokyo; also local probes at Chicago, Bonn, …

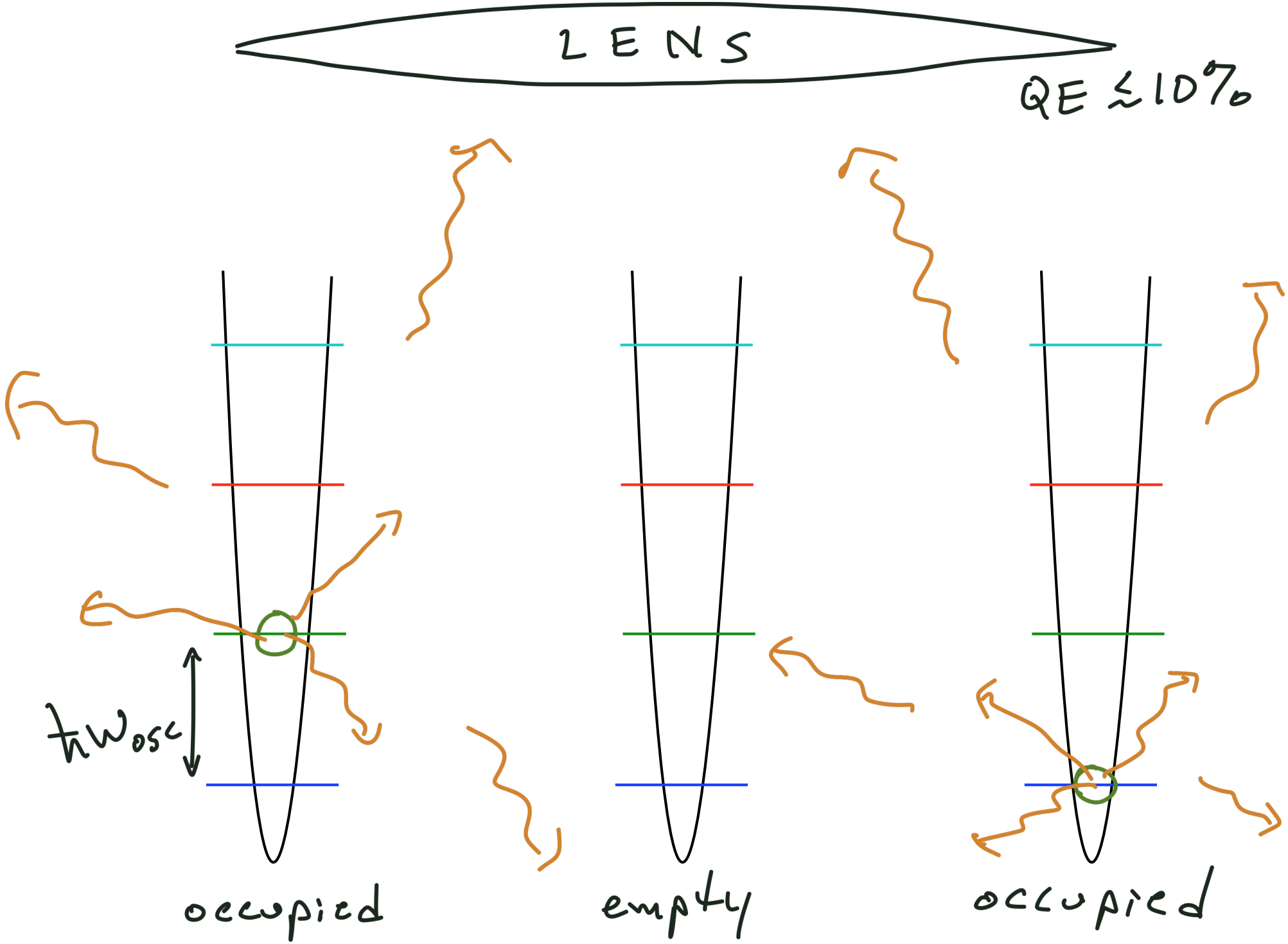
Deep lattice limit

Isolated wells, zero tunnelling

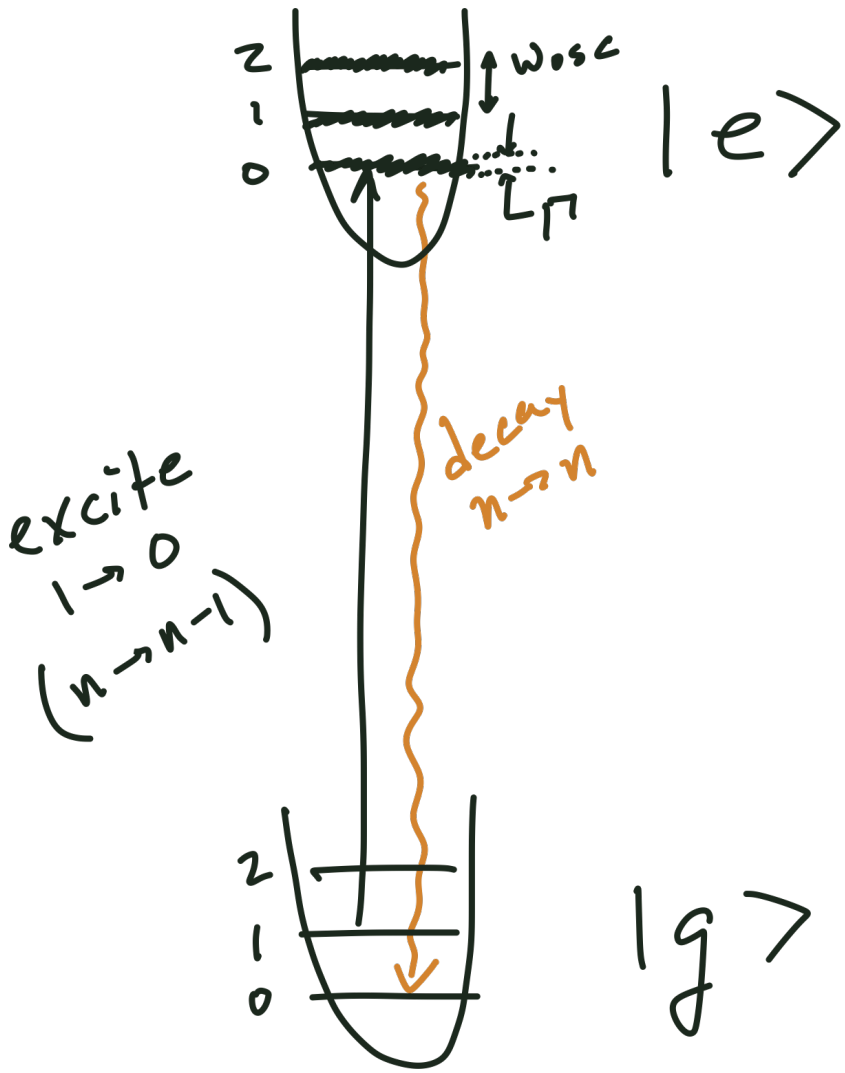
$$\hbar\omega_{\rm osc} = 2\sqrt{s}E_R$$






LENS
QE ≲ 10%
ħω_osc
occupied
empty
occupied

Sideband cooling



"resolved sideband limit":

$$\Gamma \ll \omega_{osc}$$

✓ ion traps:

$\omega_{osc} \sim$ MHz

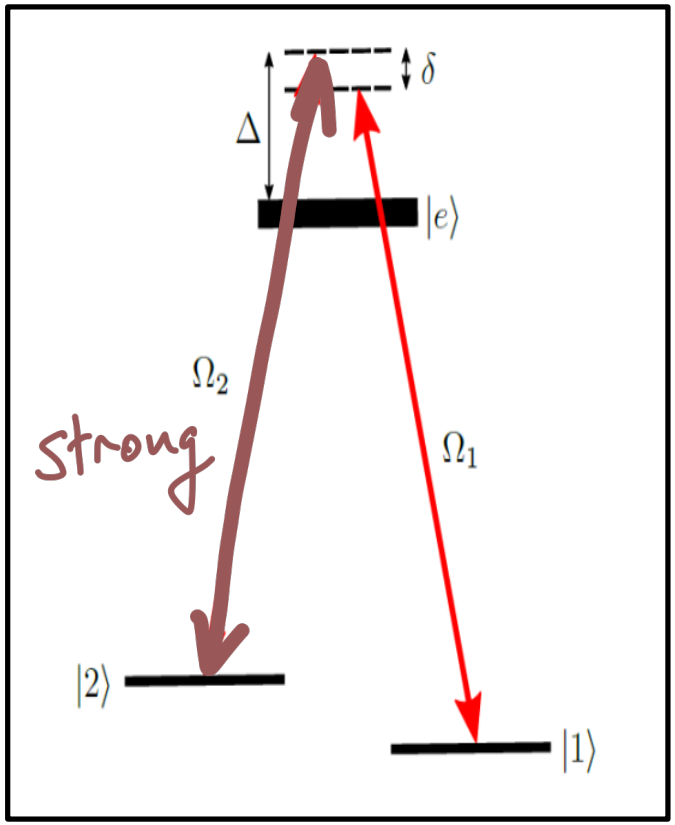
optical lattices:

$\omega_{osc} = 2\sqrt{s}\, E_R$

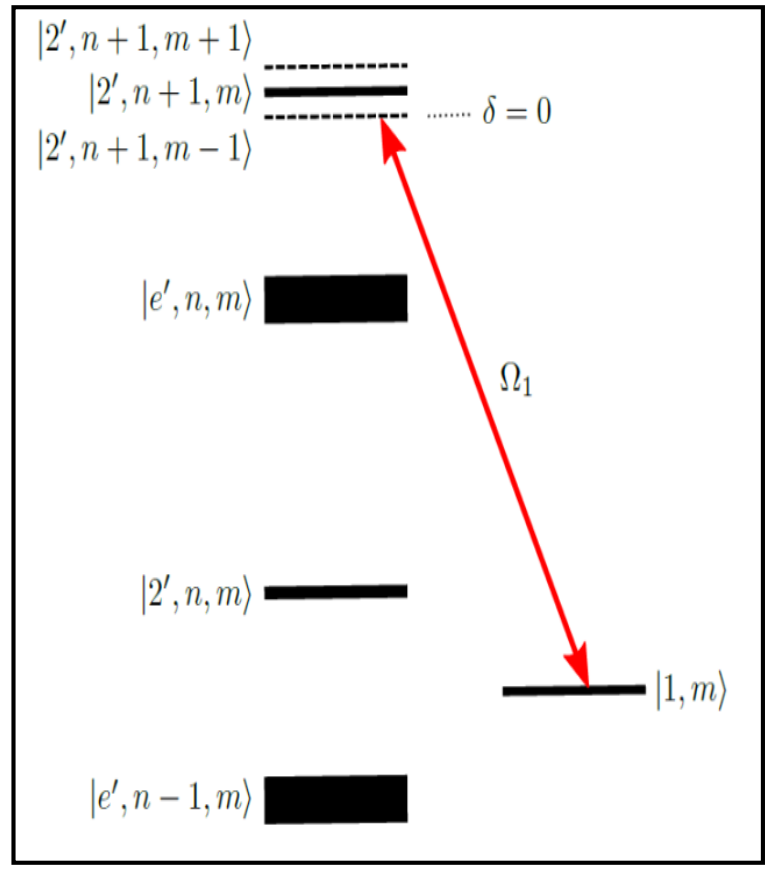
× $E_R \sim 5$ kHz

$\Gamma \sim 6$ MHz

Sideband cooling: EIT scheme

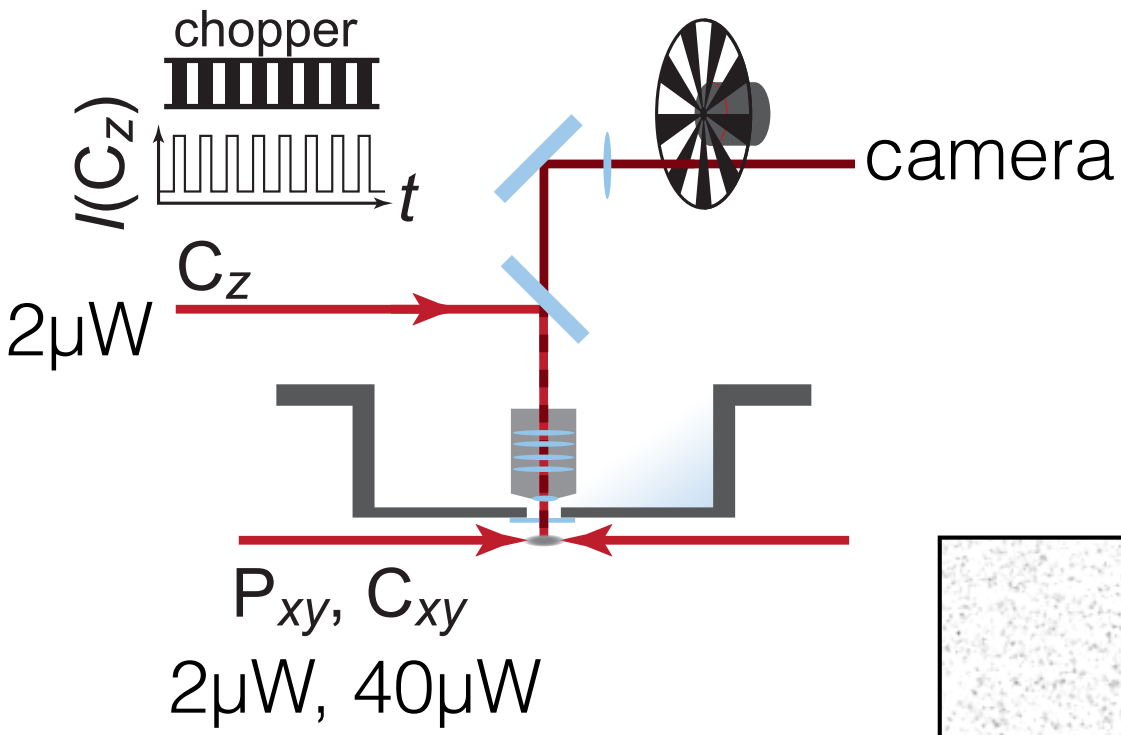


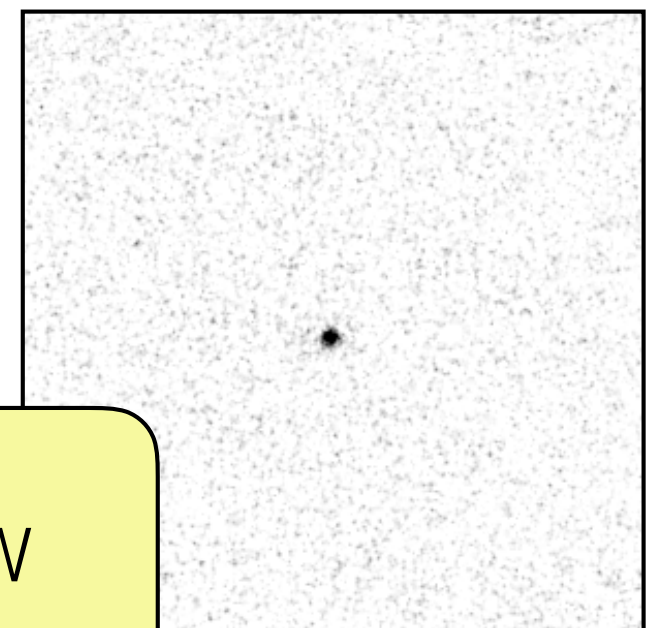
bare states



dressed with strong beam

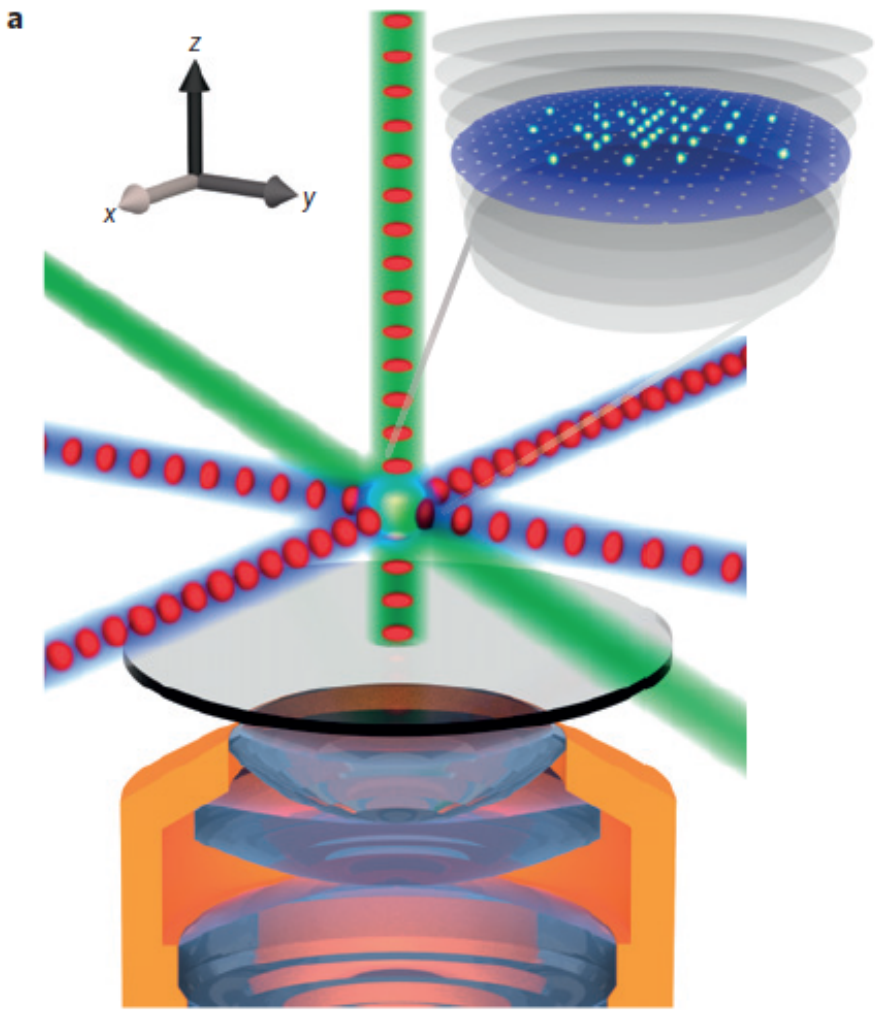
Time-modulation of vertical cooling light

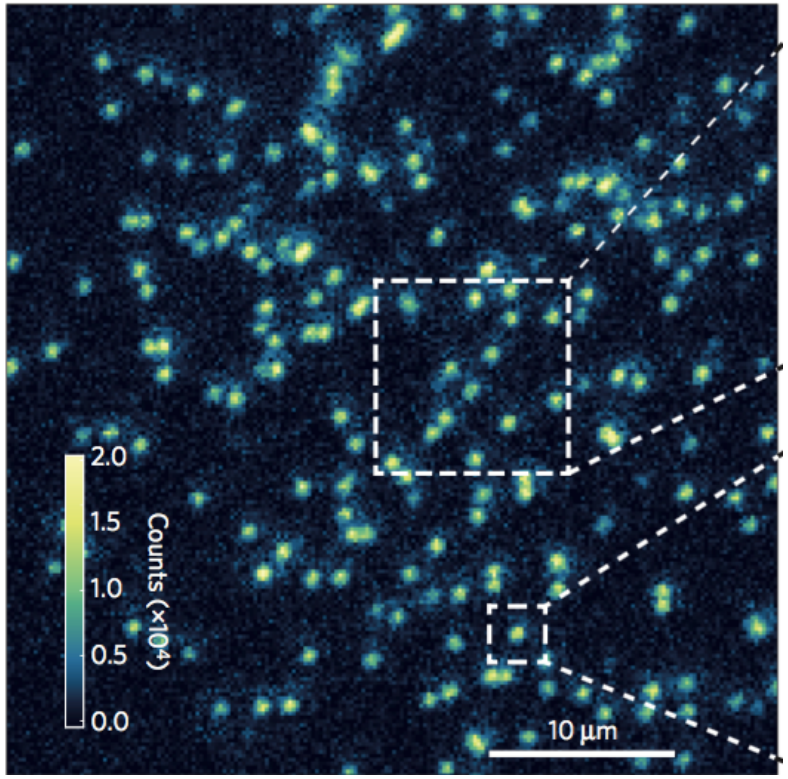




Light from a single atom: 20 aW

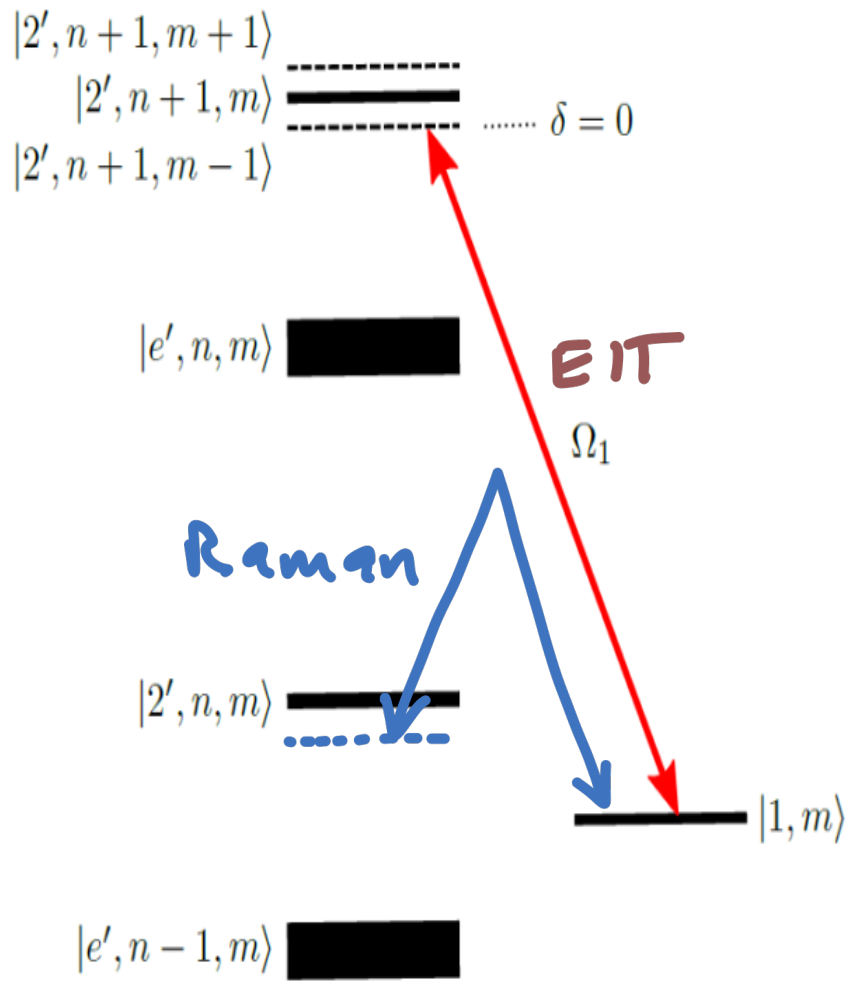
(10^{11} times weaker than Cz)



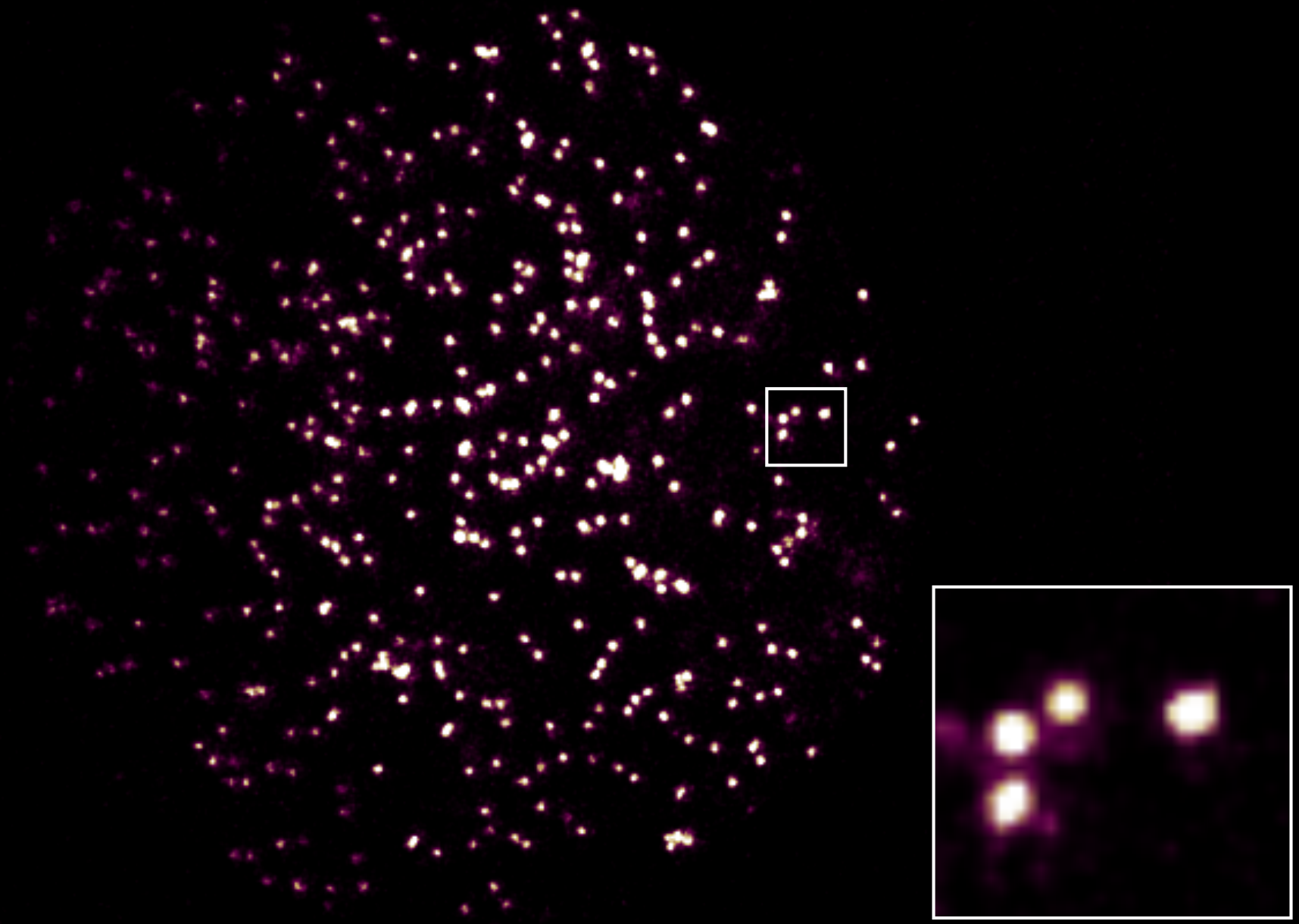


Strathclyde arXiv:1503.02648
/ Nat Phys (2015)

EIT+Raman!


$|2', n+1, m+1\rangle$
$|2', n+1, m\rangle$
$|2', n+1, m-1\rangle$
$\delta = 0$
$|e', n, m\rangle$
EIT
Ω_1
Raman
$|2', n, m\rangle$
$|1, m\rangle$
$|e', n-1, m\rangle$

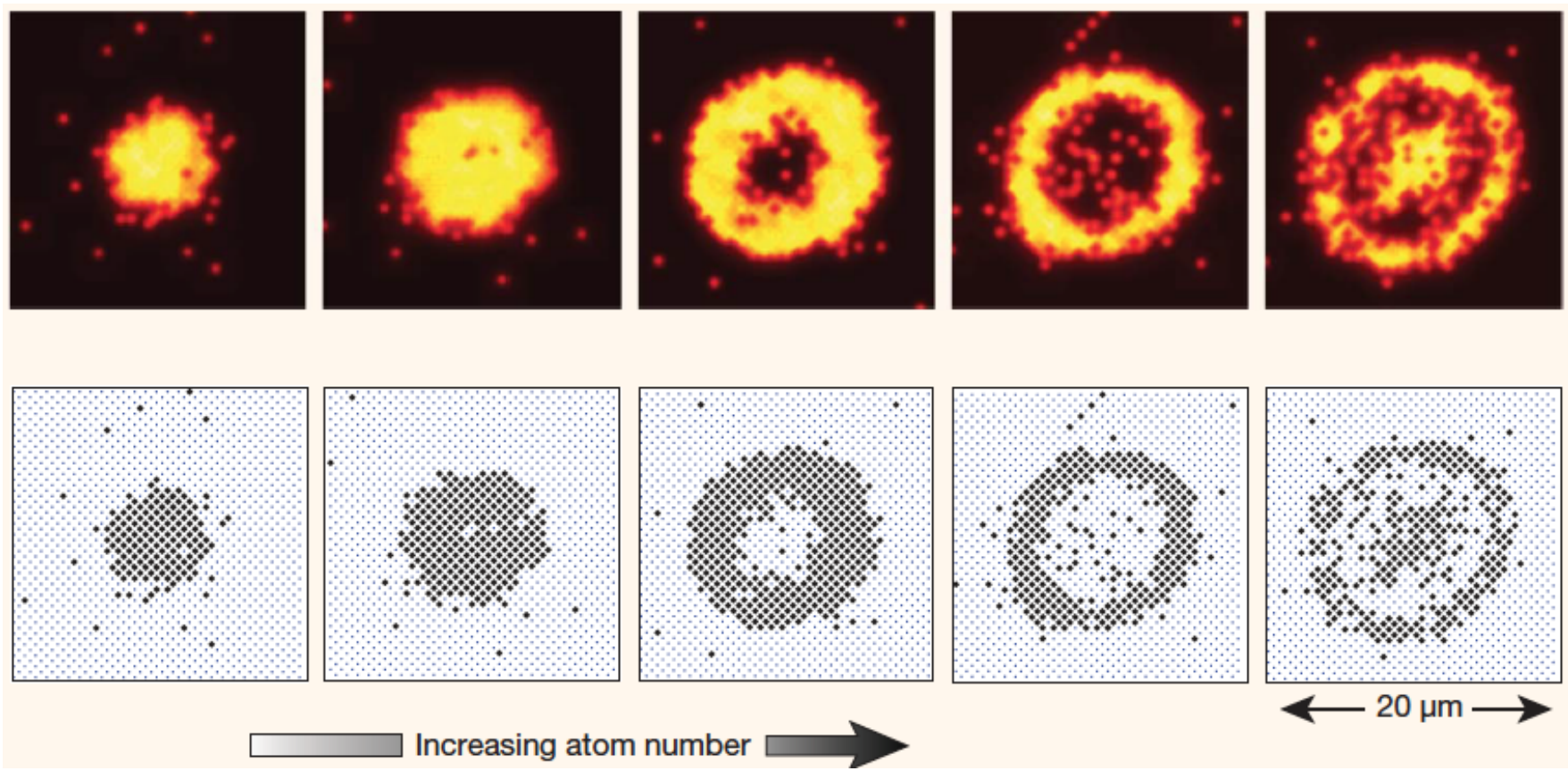
Combined EIT+Raman sideband cooling



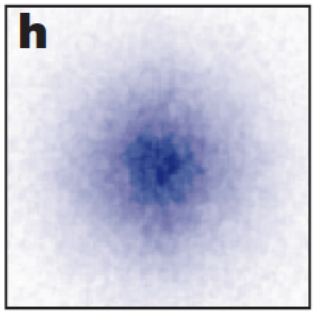
10 Sept 2016

Observation of commensurate order

Munich (2010)



cf. time-of-flight:
(no signature)



Munich (2002)

Fermionic Mott Insulator experiments:
ETHZ, Munich, Hamburg, MIT, UIUC, Harvard, Bonn, Princeton …

Bose Hubbard model

$$\hat{H}_{\mathrm{HM}} = \hat{H}_0 + \hat{H}_{\mathrm{U}}$$

Localized basis:

$\hat{H}_0 =$ **hopping to new site**

$$-t \sum_{\langle j,\ell \rangle} \hat{b}_j^\dagger \hat{b}_\ell$$

$\hat{H}_{\mathrm{U}} =$ **local interactions**

$$U \sum_{\ell} \hat{n}_\ell (\hat{n}_\ell - 1)$$

Delocalized basis:

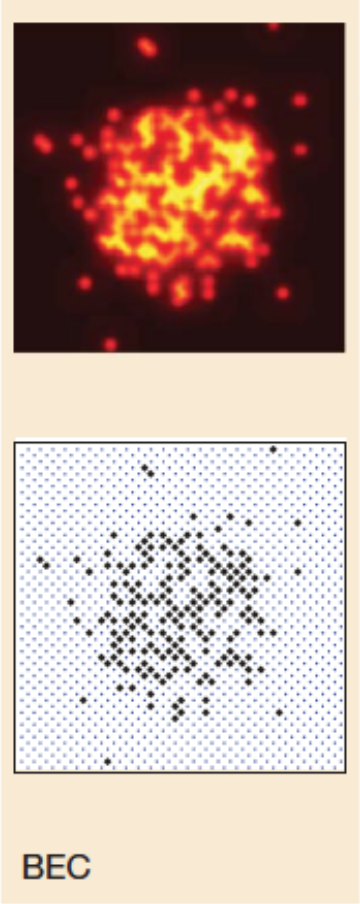
$\hat{H}_0 =$ **Bloch waves**

$$\sum_q E(q) \hat{n}_q$$

$\hat{H}_{\mathrm{U}} =$ **atom-atom scattering**

$$\frac{U}{M} \sum_{q_1,q_2,q_3} \hat{b}_{q_4}^\dagger \hat{b}_{q_3}^\dagger \hat{b}_{q_2} \hat{b}_{q_1}$$

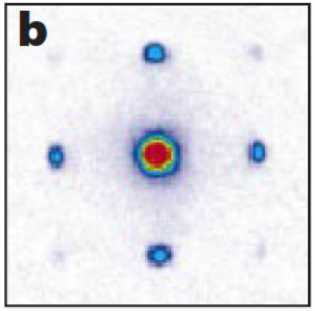
Munich (2010)



No local signature of BEC.

time-of-flight signature:

Munich (2002)



phase coherence!

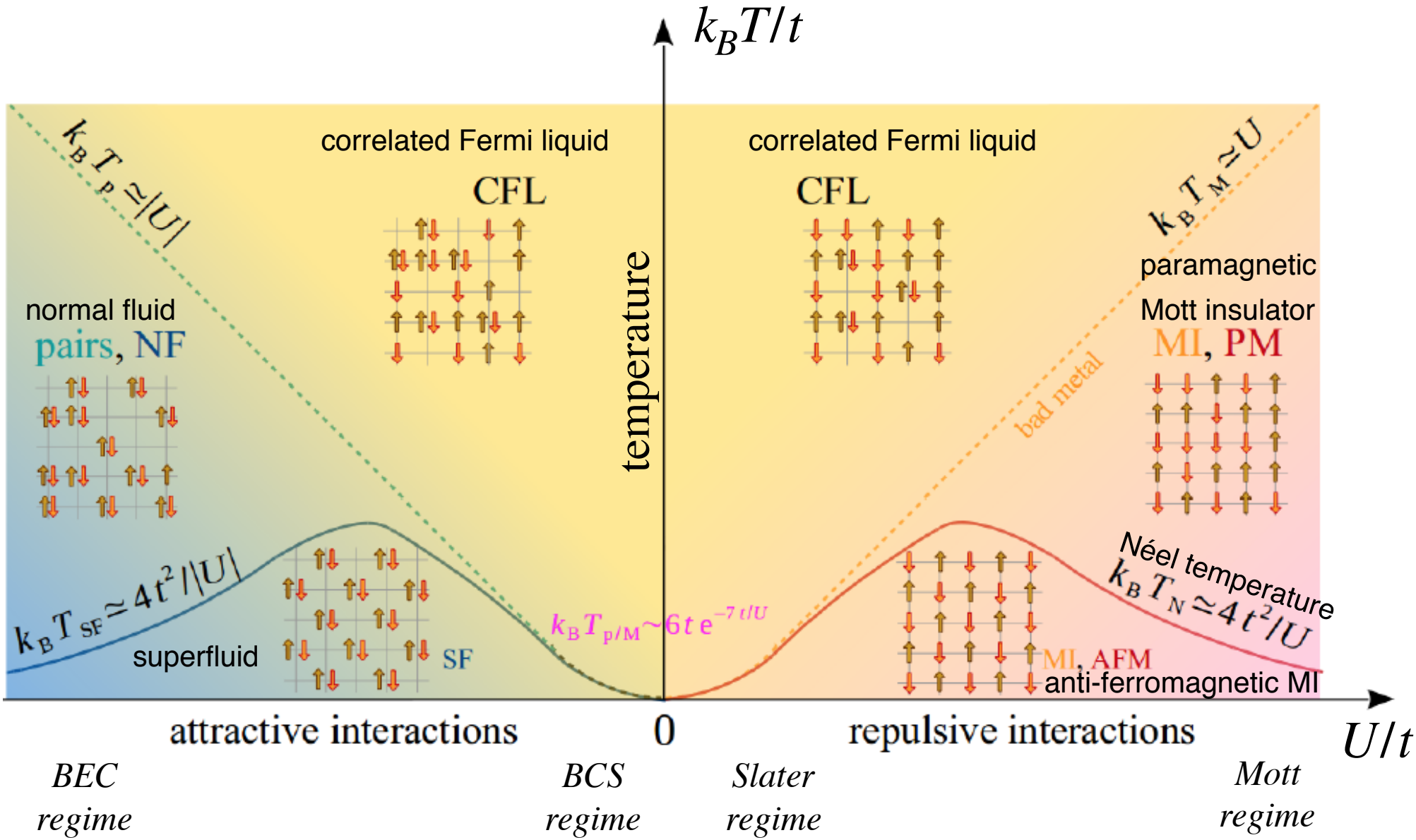
Hubbard model: observed phases, & quantum gas microscopes

Bosonic SF-to-Mott Insulator transition

Quantum Gas microscopy

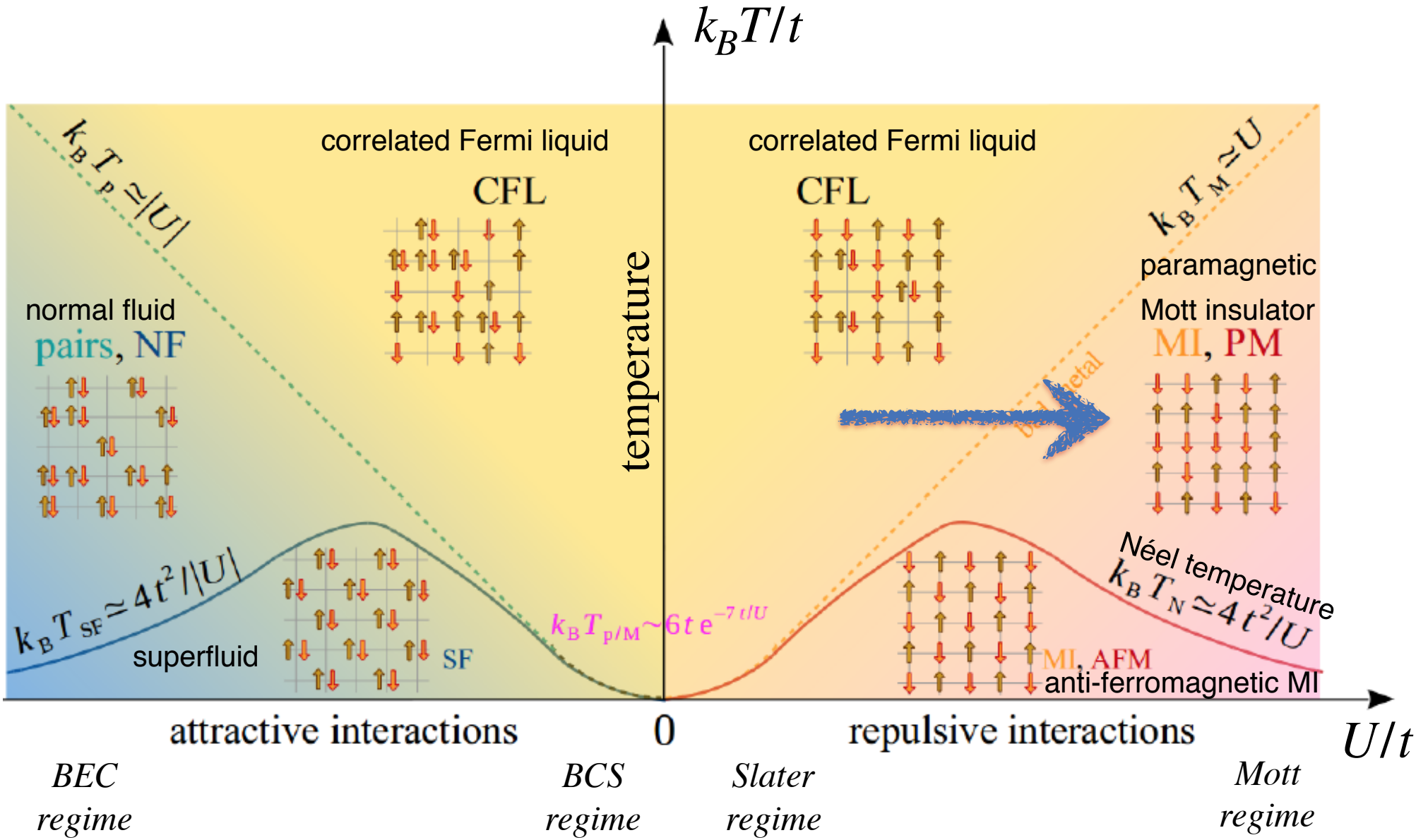
Phases of Fermions in optical lattices

Phase diagram of the Fermi-Hubbard model at half filling



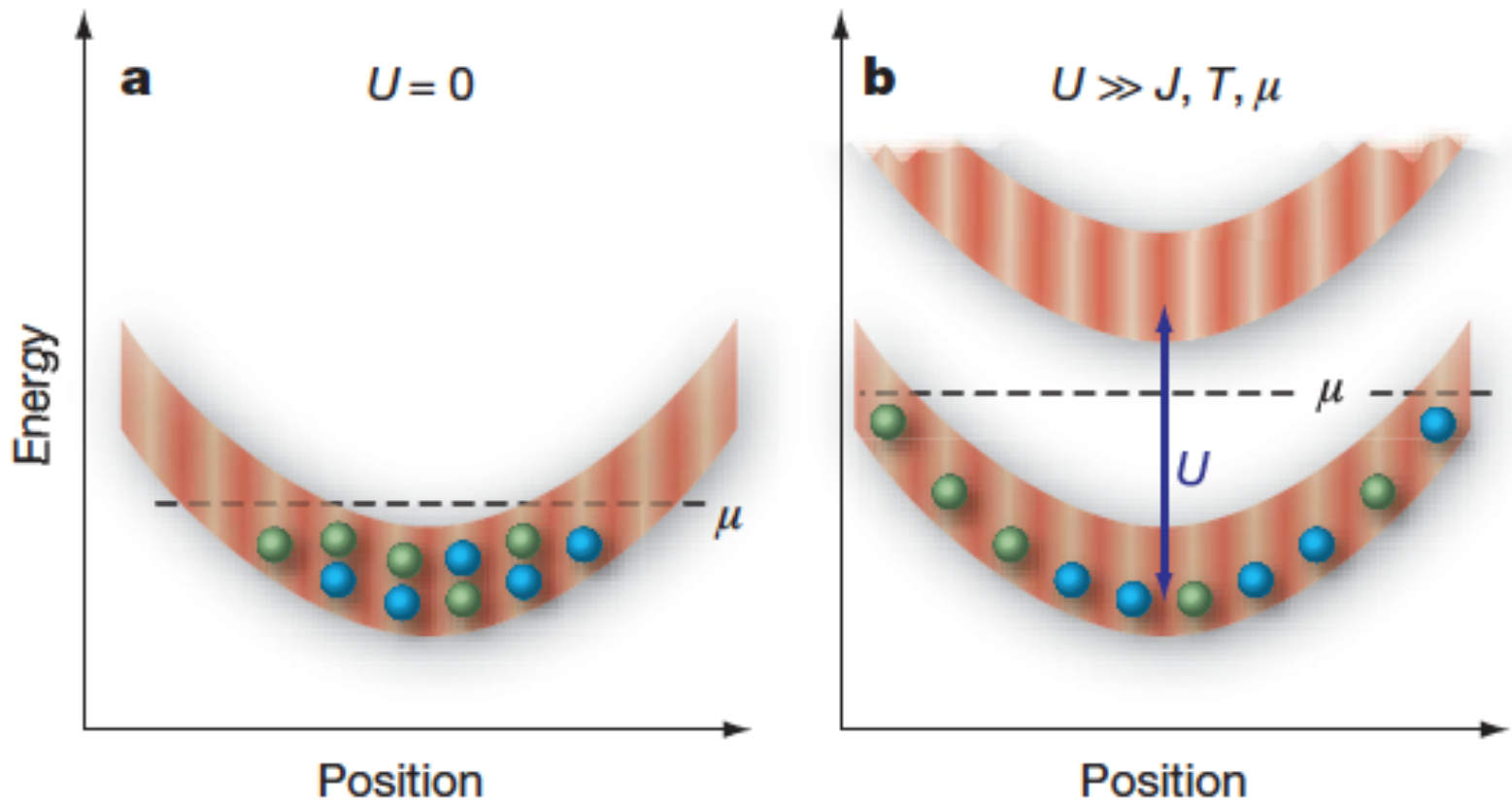
Tarruell & Sanchez-Palencia (2018)

Phase diagram of the Fermi-Hubbard model at half filling



Tarruell & Sanchez-Palencia (2018)

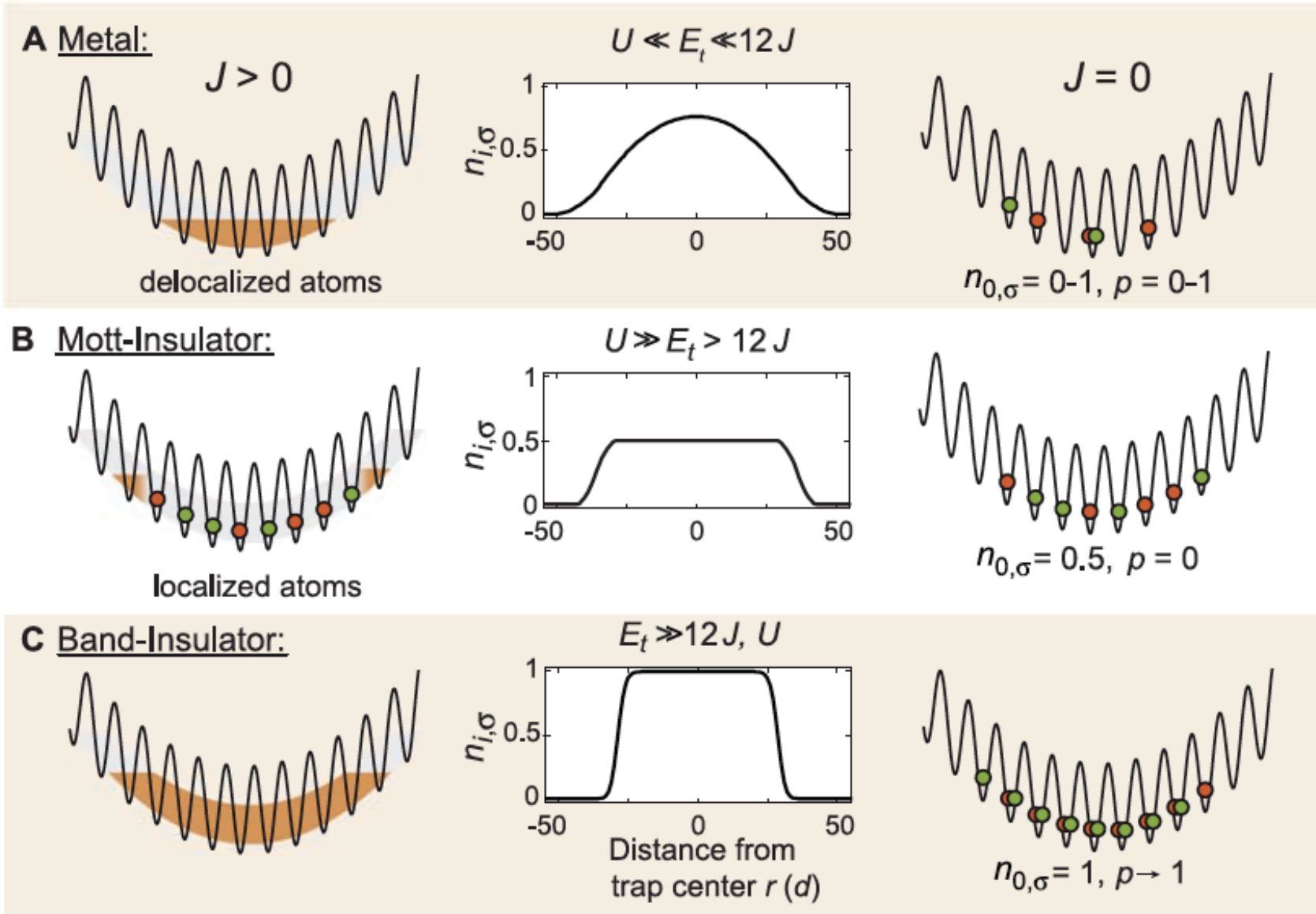
Mott Insulator in fermions



ETHZ (2008)

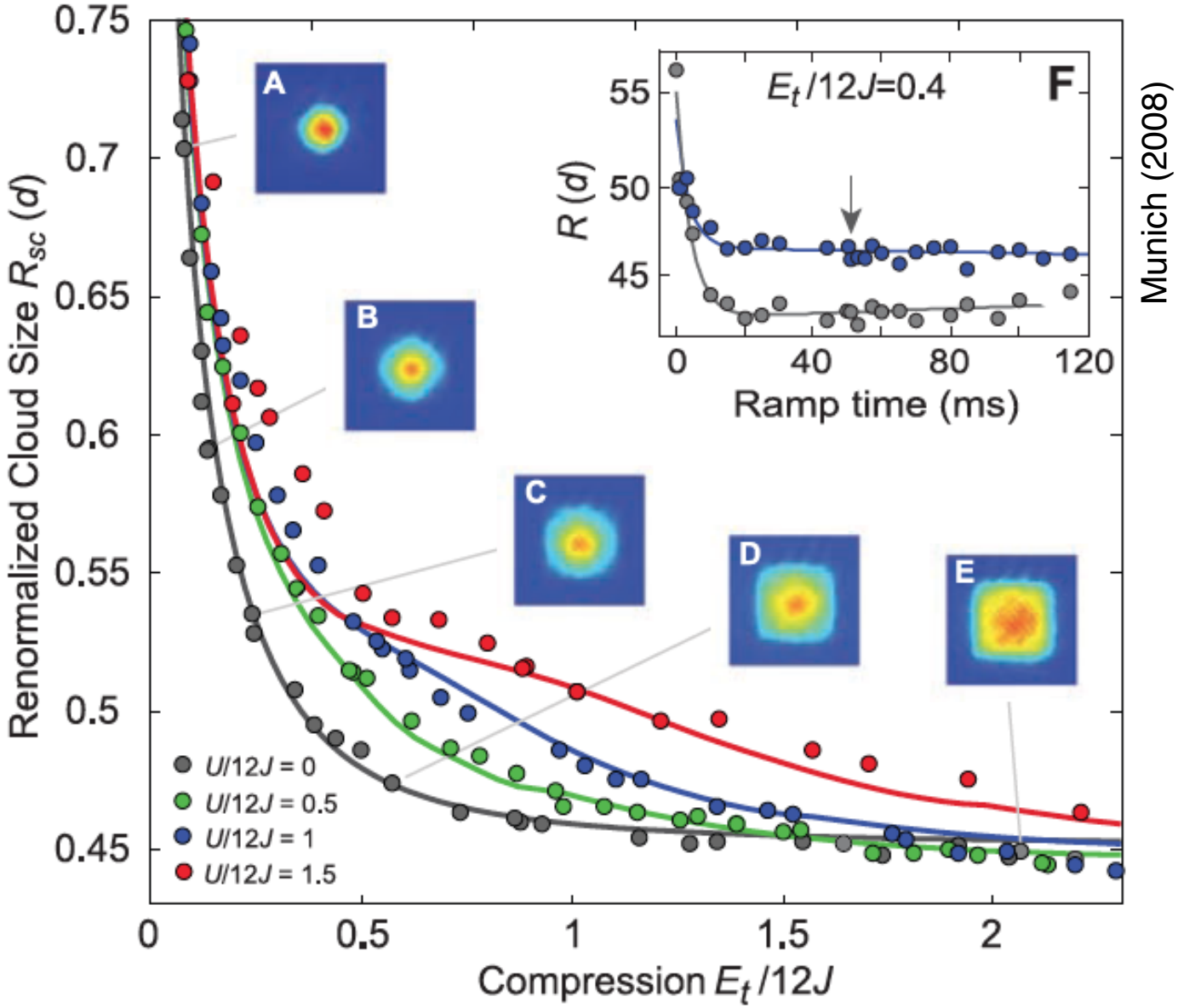
Fermionic Mott Insulator experiments:
ETHZ, Munich, Hamburg, MIT, UIUC, Harvard, Bonn, Princeton …

Insulating states of fermions



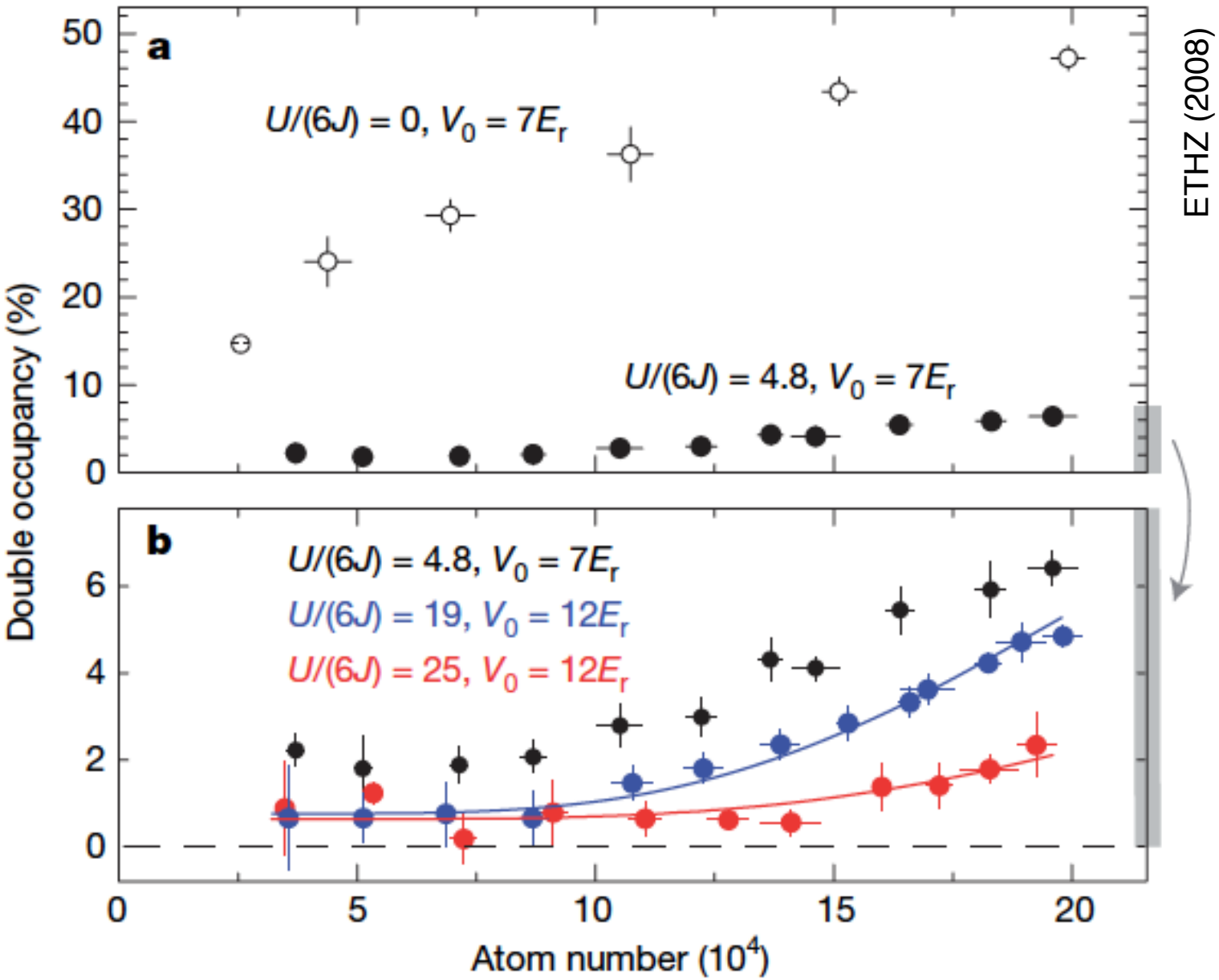
Munich (2008)

Evidence for insulating state of fermions:
(1) reduced compressibility



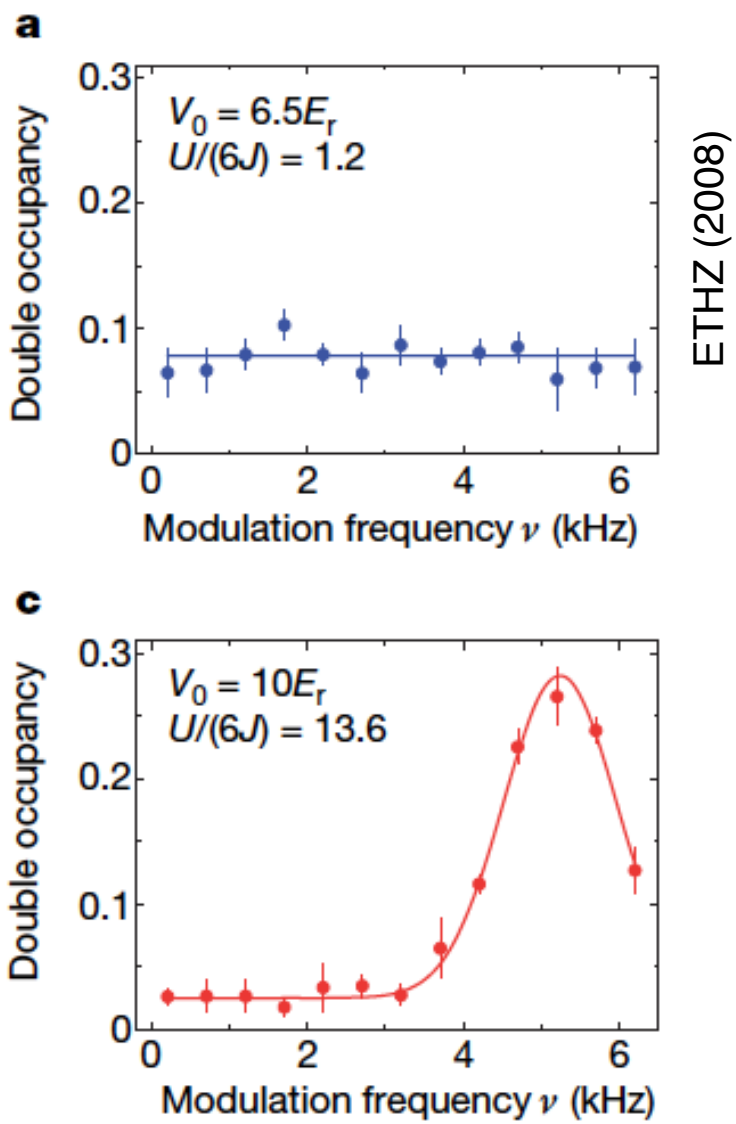
Munich (2008)

Evidence for insulating state of fermions: (2) fewer doublons

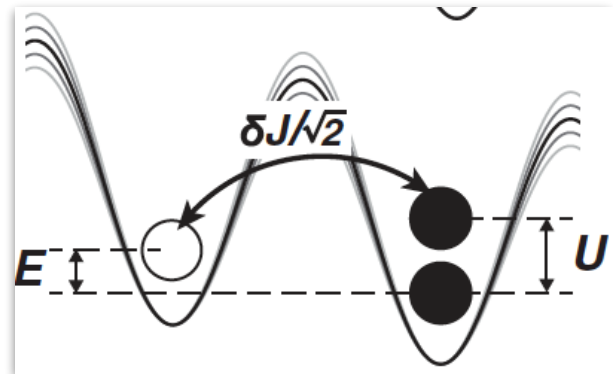


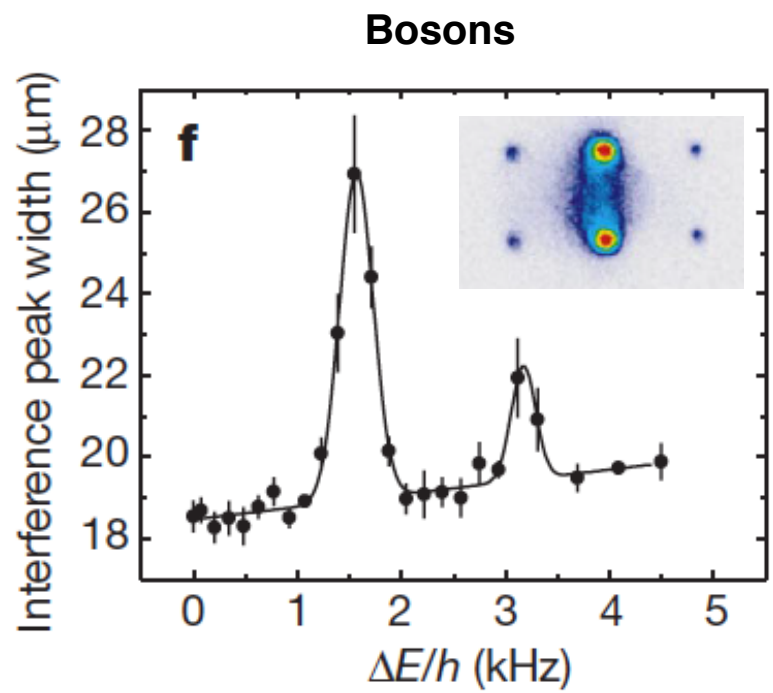
ETHZ (2008)

Evidence for insulating state of fermions: (3) Mott gap



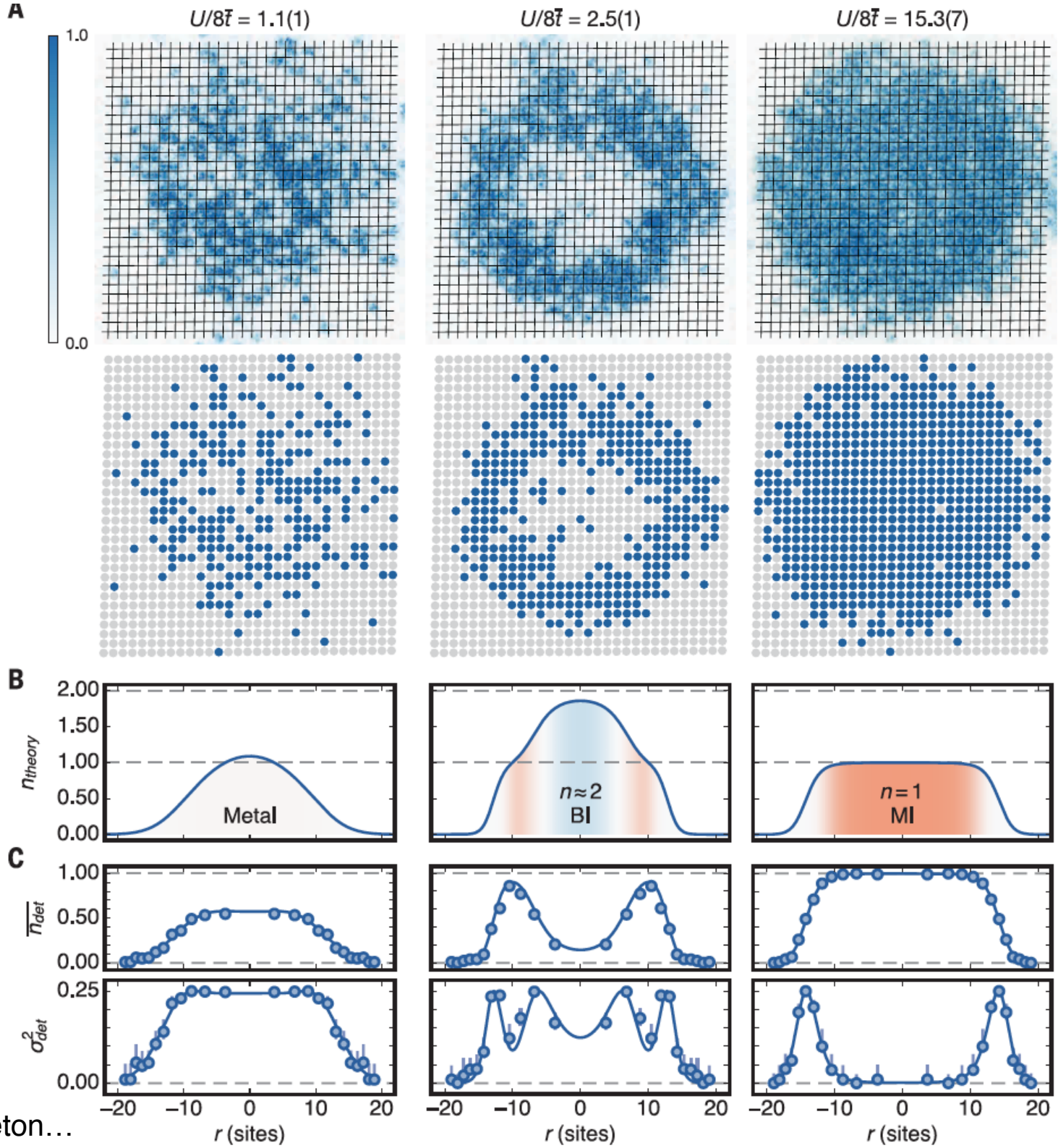
ETHZ (2008)





Munich (2002)

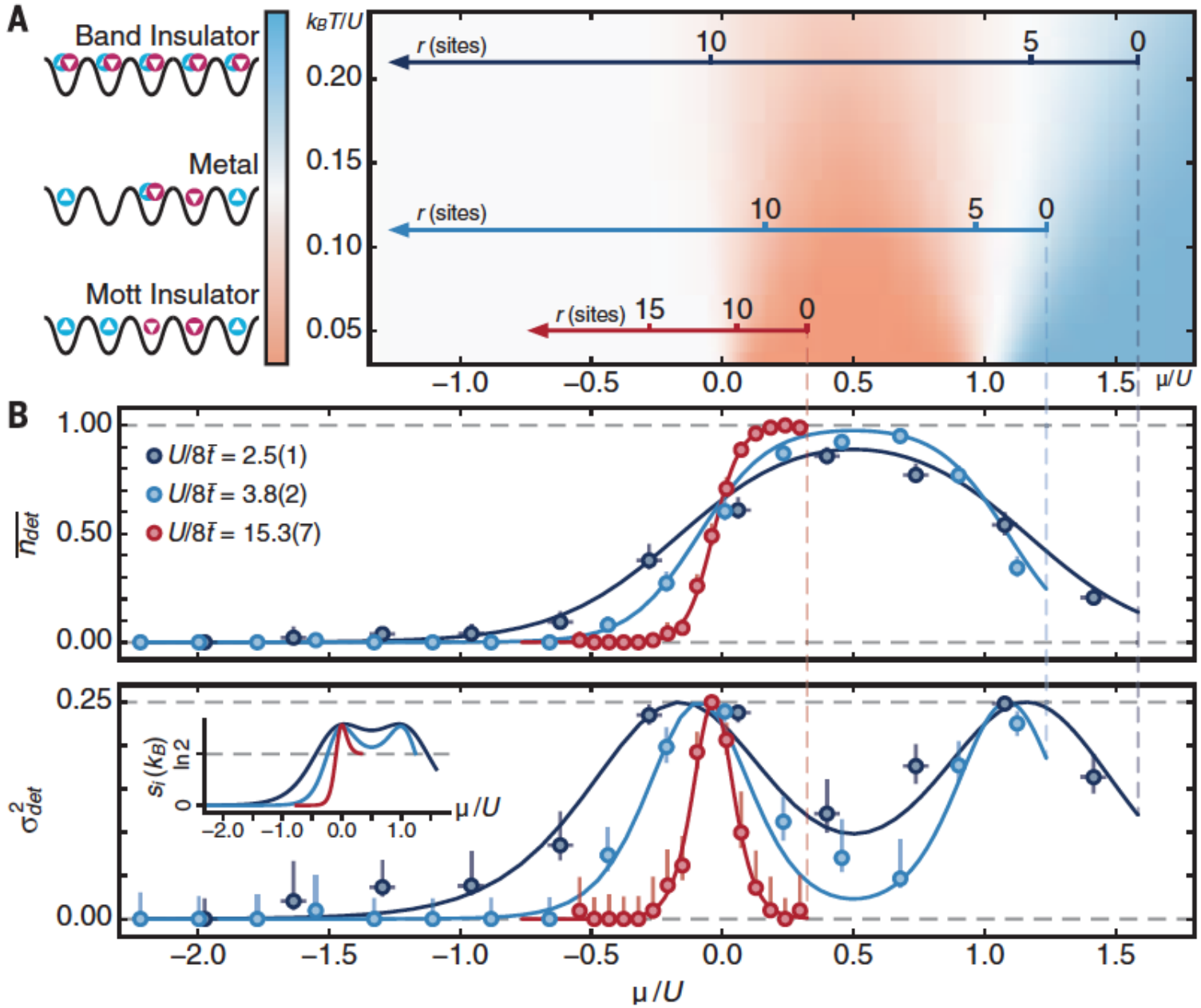
Direct observation of insulating order parameter



Harvard (2016)

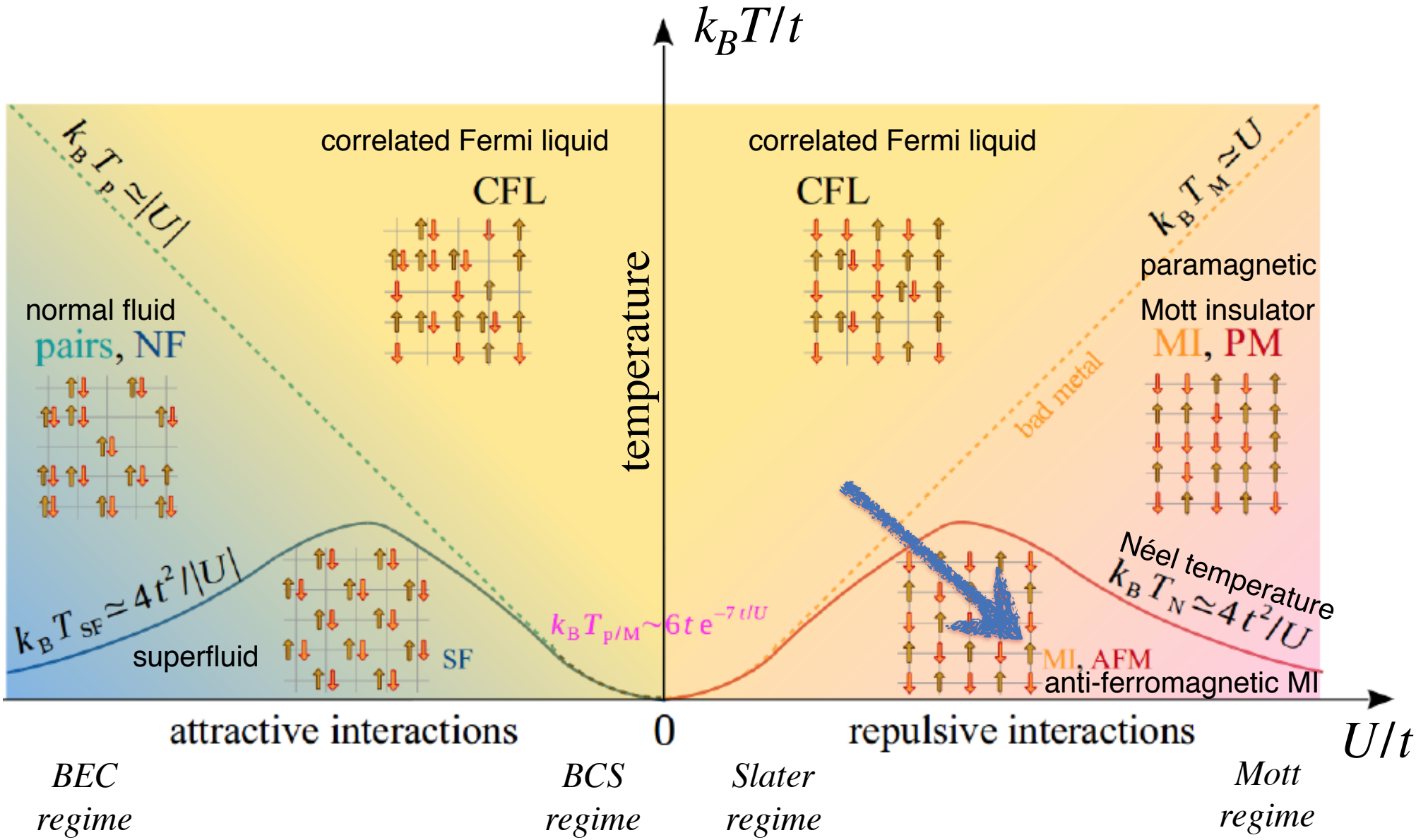
similar results:
Munich, Harvard, MIT, Princeton…

Trap effect: change local chemical potential

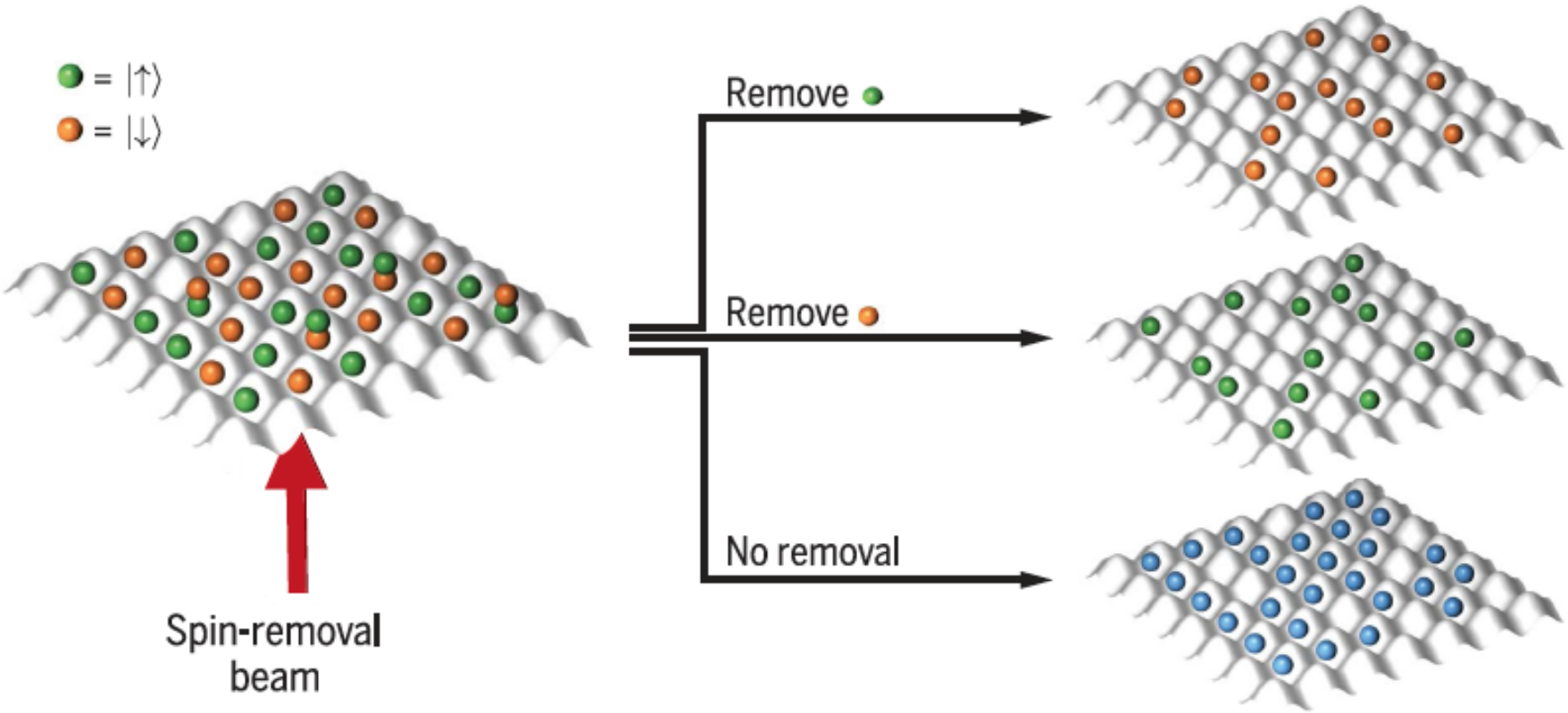


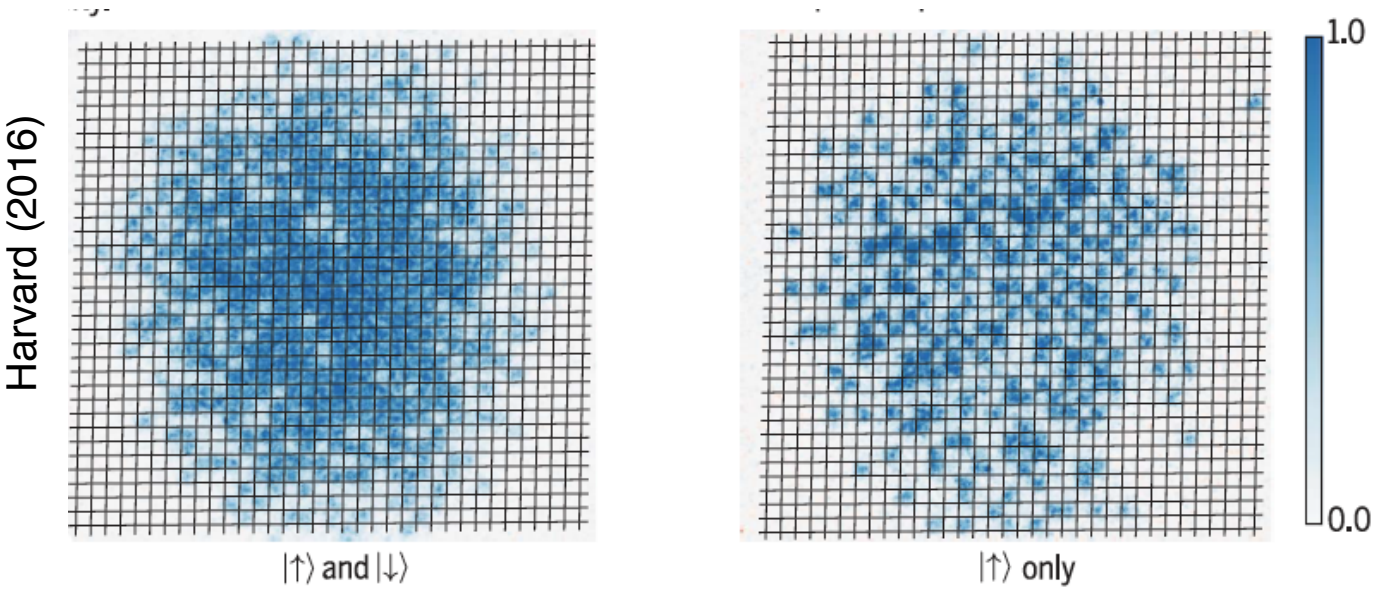
Harvard (2016)

Phase diagram of the Fermi-Hubbard model at half filling

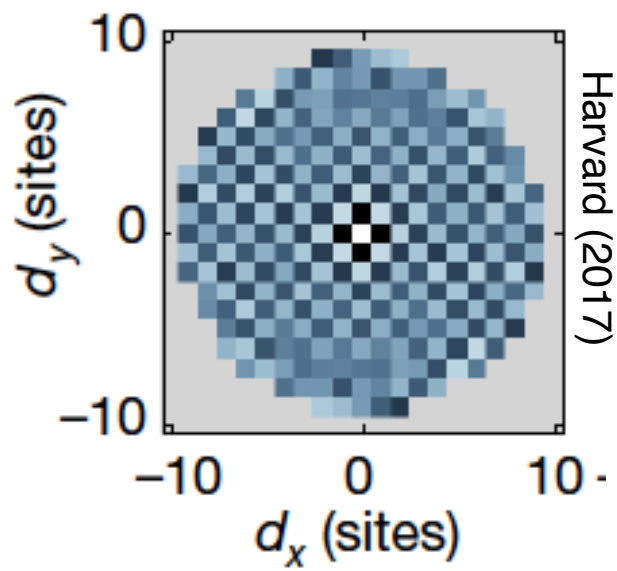


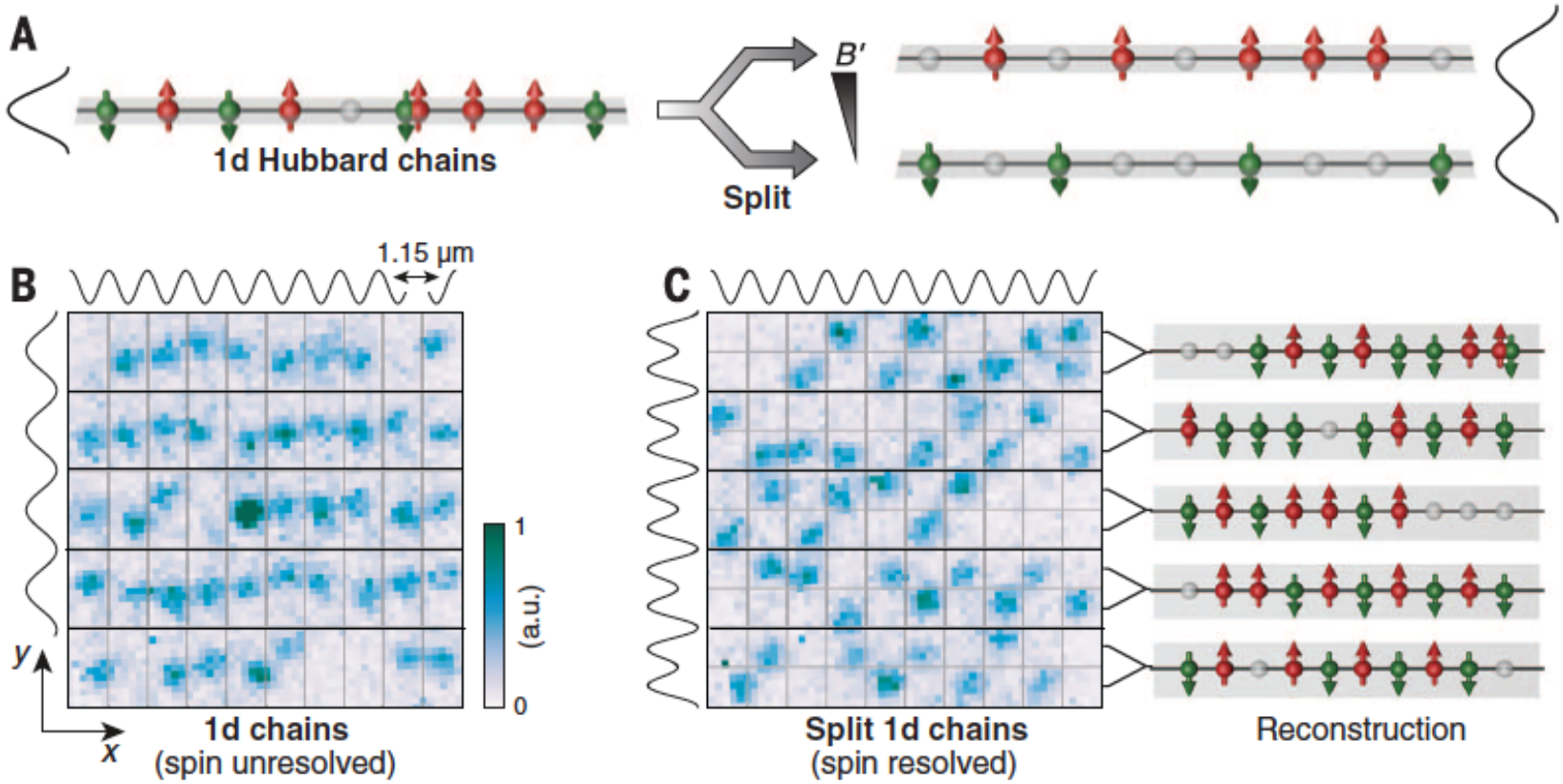
Tarruell & Sanchez-Palencia (2018)


= |↑⟩
= |↓⟩
Remove
Remove
No removal
Spin-removal beam


Harvard (2016)
|↑⟩ and |↓⟩
|↑⟩ only
1.0
0.0

colder: AFM!


Harvard (2017)
10
0
−10
−10
0
10
d_y (sites)
d_x (sites)



Munich (2016)

[PS — see Joannis Koepsell's poster on imaging polarons in 2D with this technique!]

Summary, Topic 3

- The Hubbard Model (HM) is the minimal model for interacting fermions in an optical lattice
- HM in localized basis = hopping + on-site interaction
  HM in quasi-momentum basis = KE + scattering
- Rapid progress in quantum simulation of the Fermi Hubbard model, driven by new quantum in-situ detection techniques
- Observations of Mott insulators, band insulators, anti-ferromagnetism, and more.
- Frontiers: even lower temperature; away from half-filling; and spin imbalance.

ÉCOLE DE PHYSIQUE
des HOUCHES

Fermions in Optical Lattices

- ✓ Introduction
- ✓ Fermions, statistics, & exchange
- ✓ Matter waves in crystals of light
- ✓ The Hubbard model
- Transport