ÉCOLE DE PHYSIQUE
des HOUCHES

Predoc School on
Ultracold Fermions

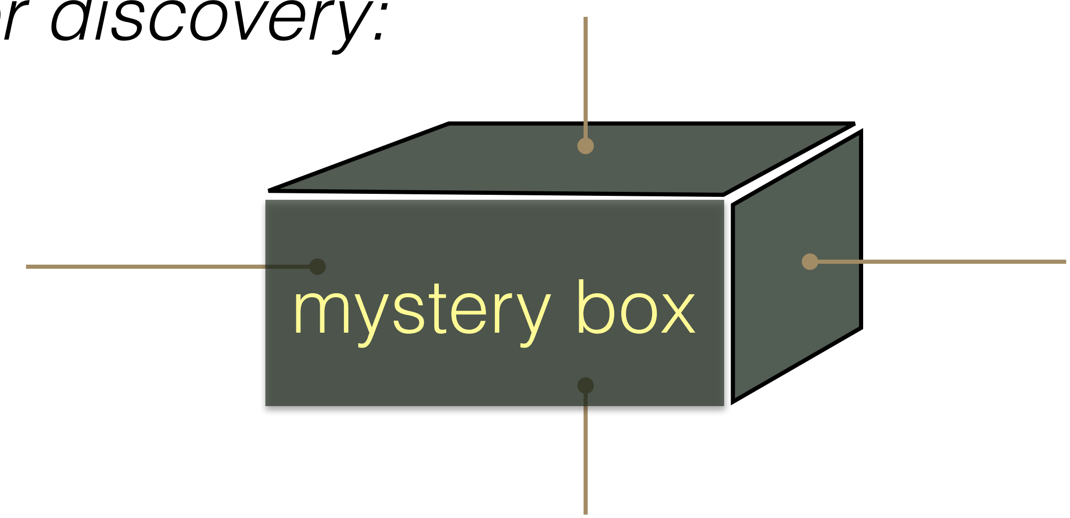
Fermions in Optical Lattices

8-9 October 2018
Joseph H Thywissen
University of Toronto

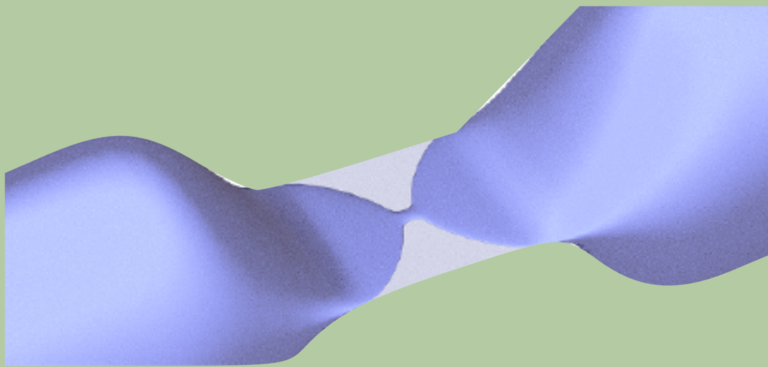
Transport phenomenology

- Metal
- Superconductor
- Insulator
- Quantum Hall State
- …

A tool for discovery:



channel

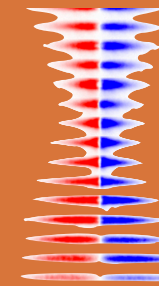


eg: EHTZ; EPFL; NIST/JQI

see review by Brantut, Esslinger, et al.
J Phys. Condens. Mat. (2017)

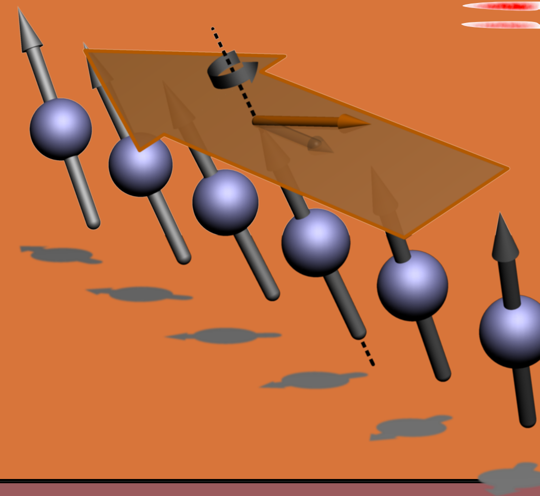
trap

(especially: spin transport)



Roati, Zwierlein *et al.*

MIT
Cambridge
Rice
Toronto
LENS
LKB
....



Trotzky, JHT *et al.*

disorder

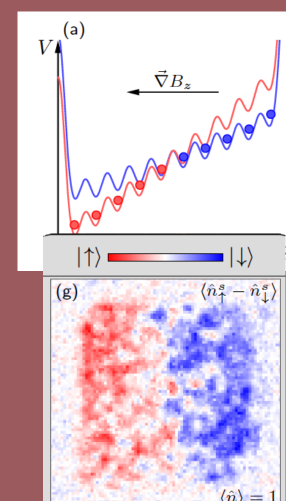
eg: MBL

Palaiseau
LENS
UIUC
Munich
...

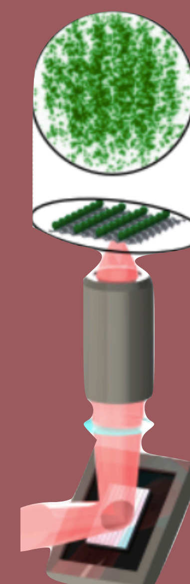
Gross, Bloch, *et al.*

lattice

LENS
ETHZ
UIUC
Munich
MIT
Princeton
Toronto
....



Zwierlein *et al.*



Schauss, Bakr *et al.*

How do electrons move through materials?

$$\frac{\text{Response}}{\text{Force}} = \text{Conductivity}$$

millennium of physics

"motion requires effort"

$$v \propto F$$

inertia

$$\frac{dv}{dt} \propto F$$

open systems

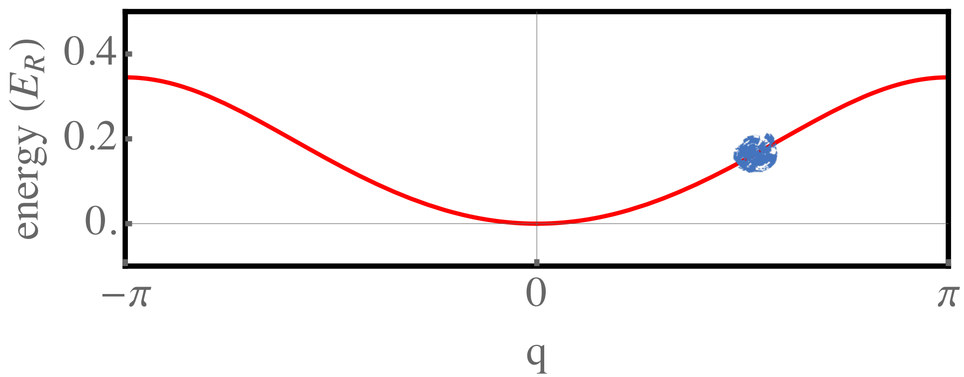
v=0 preferred

dissipation

Galilean invariance

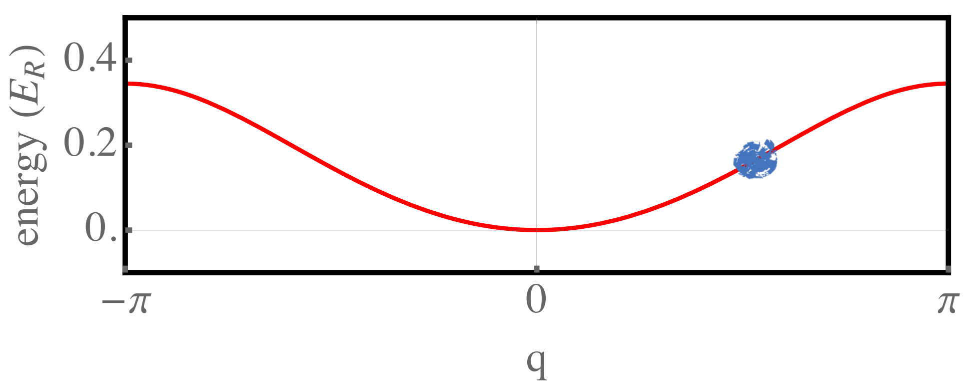
“F=ma” in a lattice?

A weak external force changes quasi-momentum q.



$$q(t) = q(0) + (Fa_L/\hbar)t$$

…which is periodic:



“Bloch oscillations”

$$\omega_B = Fa_L/\hbar$$

Wave packet dynamics

For a wave packet localised in q,
displacement occurs at group velocity,

$$v = \frac{a_L}{\hbar}\frac{\partial}{\partial q}\mathcal{E}_\alpha(q)$$

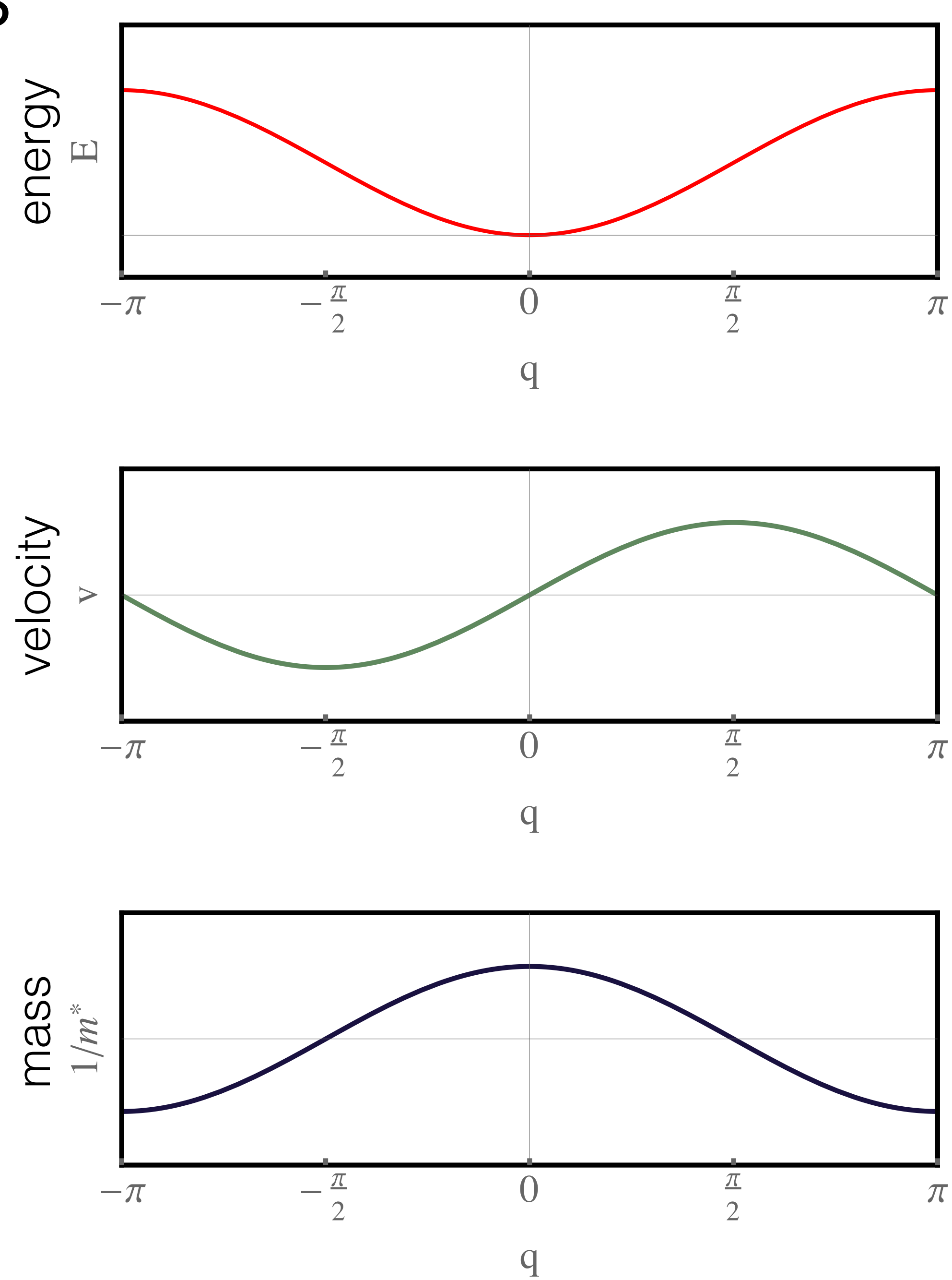
acceleration:

$$a = \frac{dv}{dt} = \frac{\partial v}{\partial q}\frac{dq}{dt} \equiv \frac{F}{m^*}$$

gives effective mass,

$$\frac{1}{m^*_\alpha(q)} = \frac{a_L^2}{\hbar^2}\frac{\partial^2}{\partial q^2}\mathcal{E}_\alpha(q)$$

band α , wave-packet at q



see for example J. E. Sipe, *PRA* **85**, 053412 (2012), & references therein.

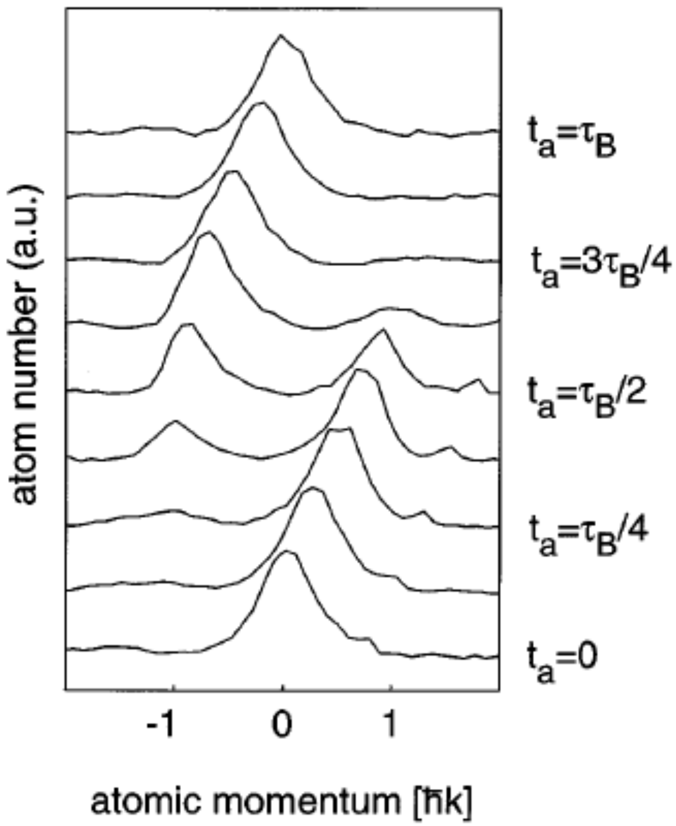
Observation of Bloch oscillations

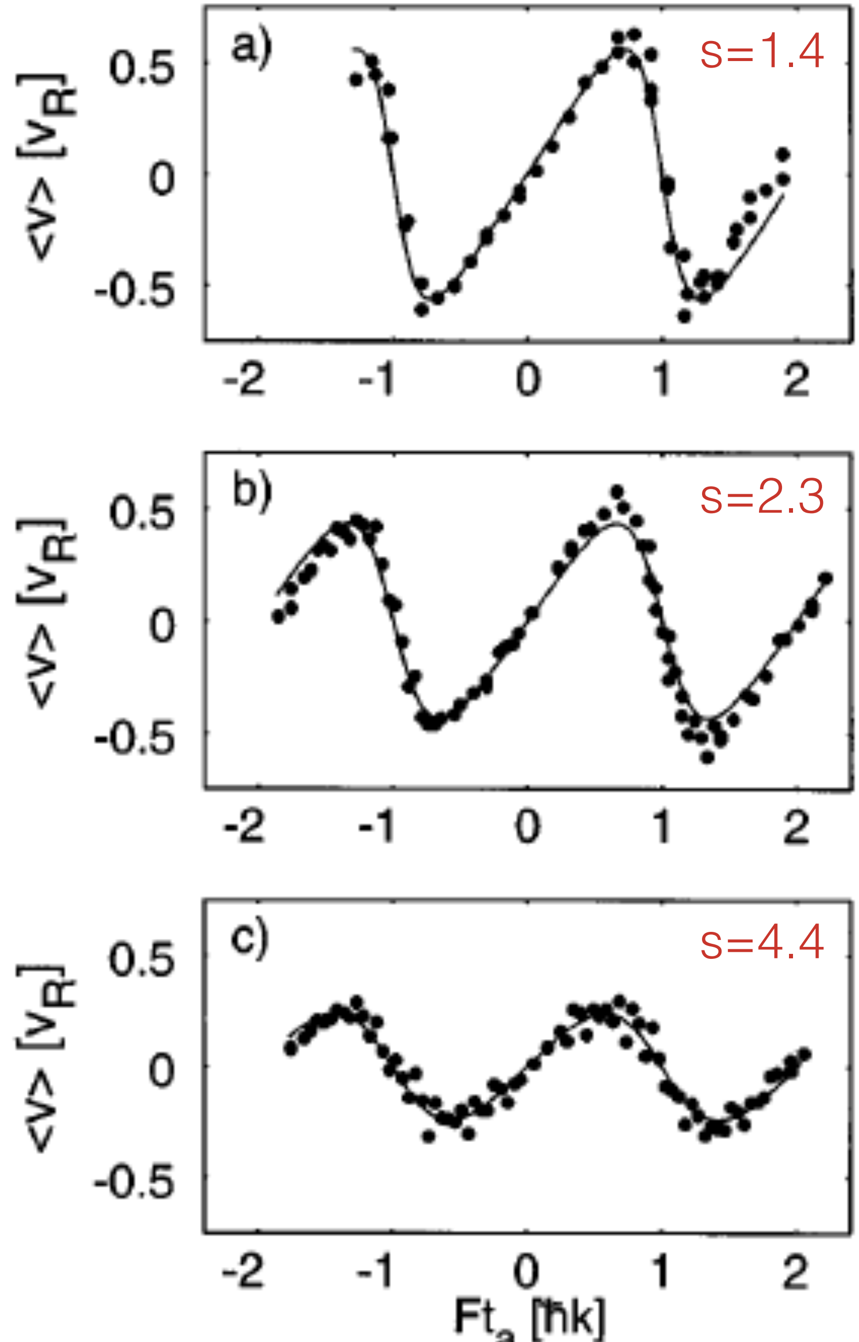
C. Salomon et al., *PRL* **76**, 4508 (1996)

-Cs in 852nm standing wave
-Time-of-flight imaging

$$\tau_B = \frac{2\hbar k_L}{|F|}$$

$$\sim 8\,\mathrm{ms}$$





Bloch period ~600 fs in semiconductor superlattices (observed by THz radiation)
Never observed in natural crystal (Bloch time longer than defect sc. time)

Observation of Bloch oscillations

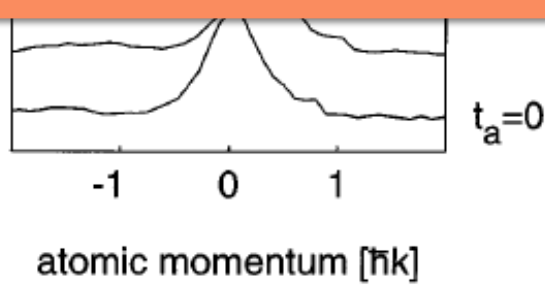
C. Salomon et al., *PRL* **76**, 4508 (1996)

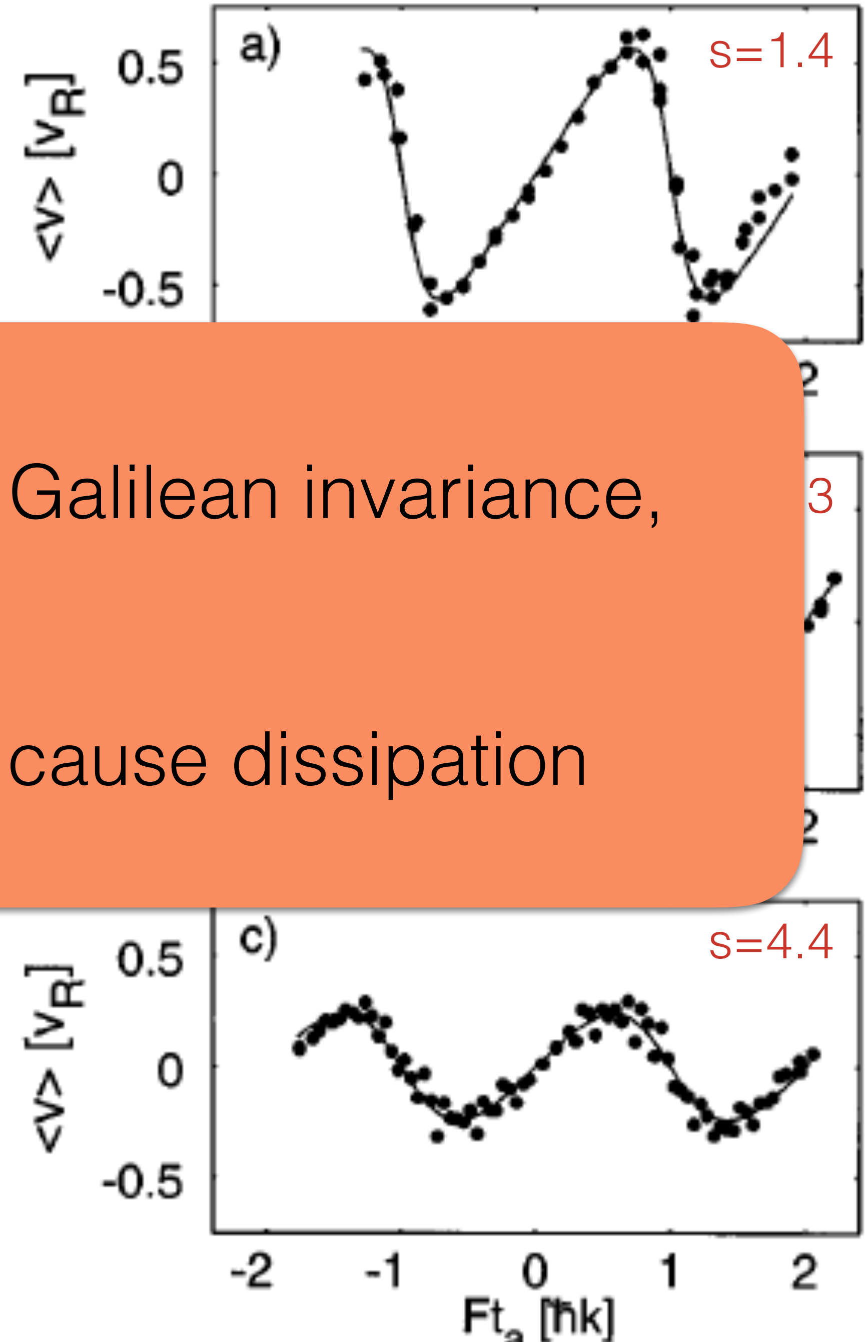
-Cs in 852nm standing wave
-Time-of-flight imaging

$\tau_B =$

$\sim$

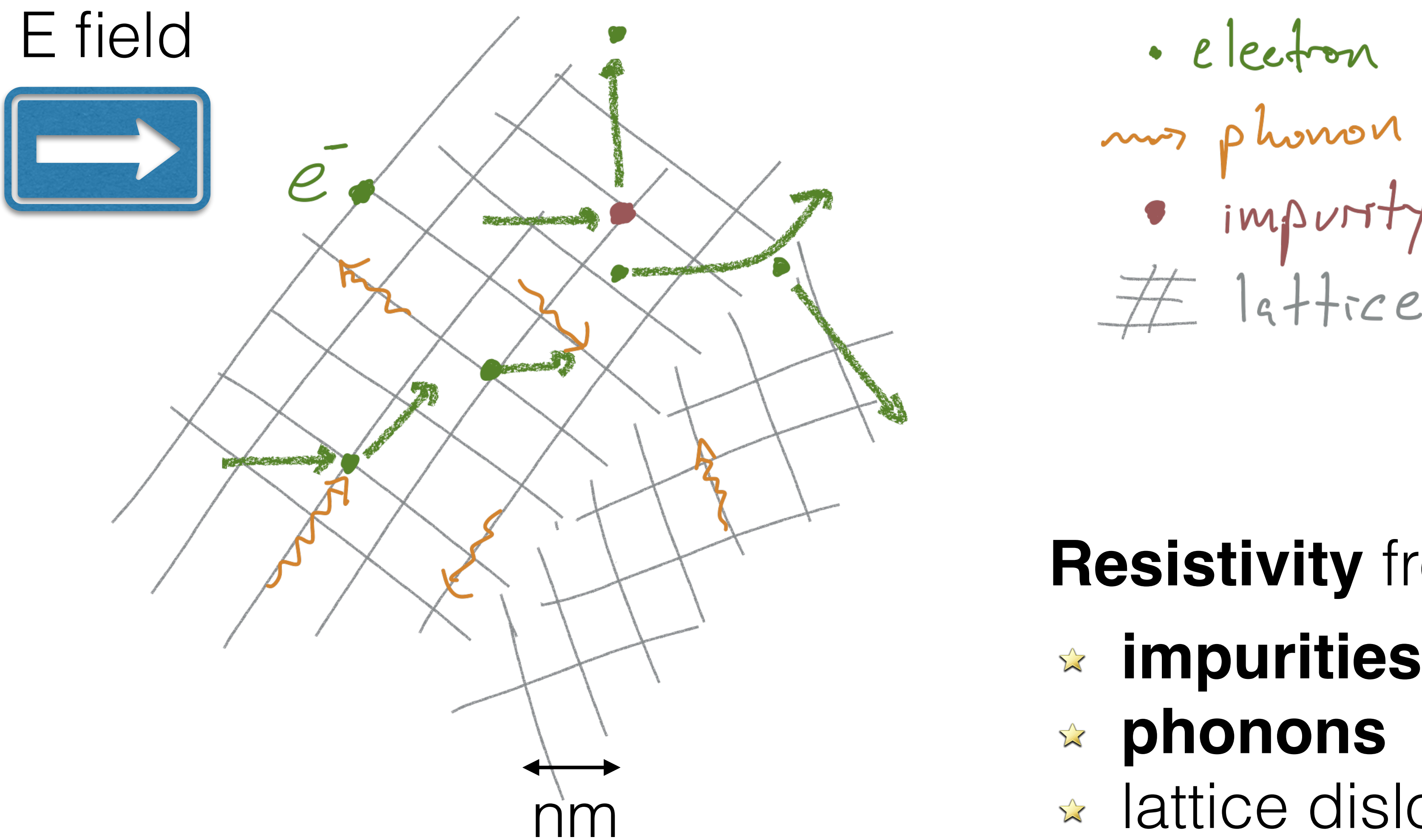
- Lattice already breaks Galilean invariance, making v=0 “special”
- Lattice alone does not cause dissipation





Bloch period ~600 fs in semiconductor superlattices (observed by THz radiation)
Never observed in natural crystal (Bloch time longer than defect sc. time)

electron transport in a metal

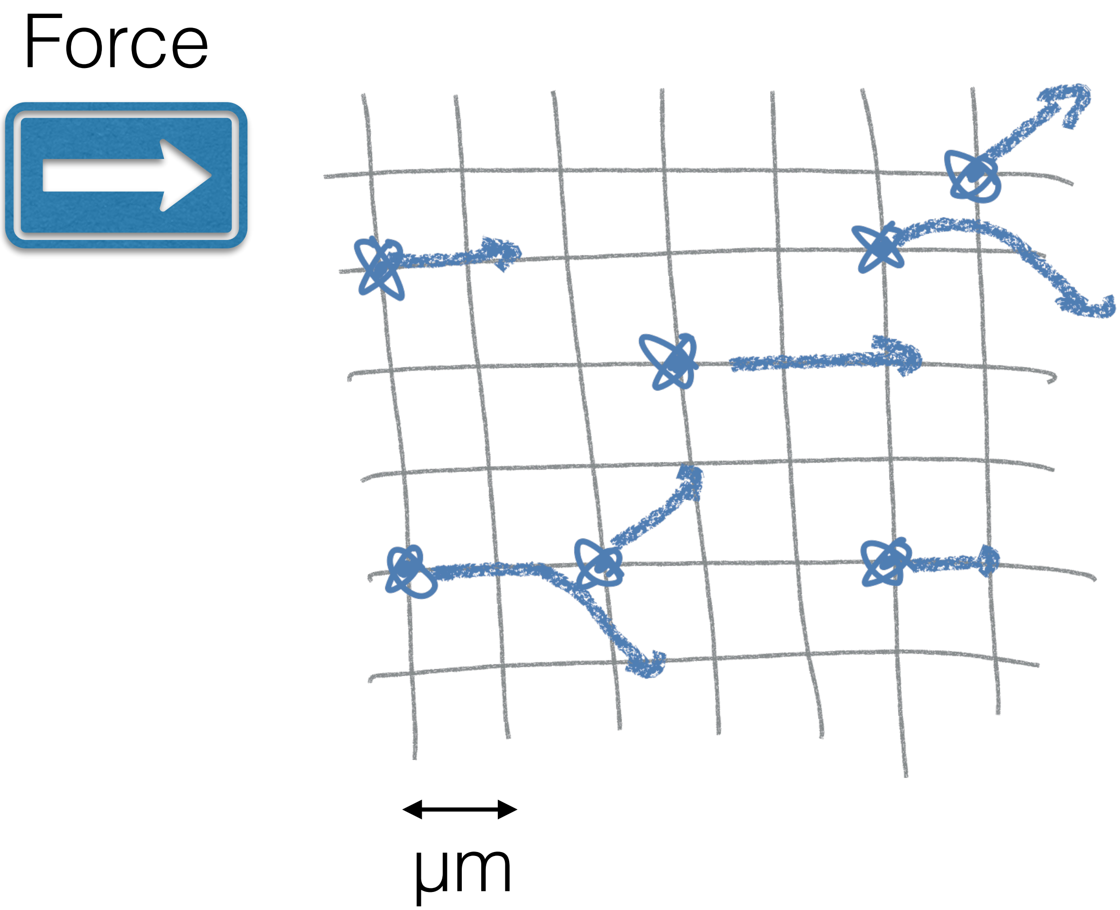


Resistivity from

- **impurities**
- **phonons**
- lattice dislocations
- particle scattering

conductivity of atoms in an optical lattice

a “perfect” crystal:
-no defects, impurities
-inflexible: no phonons

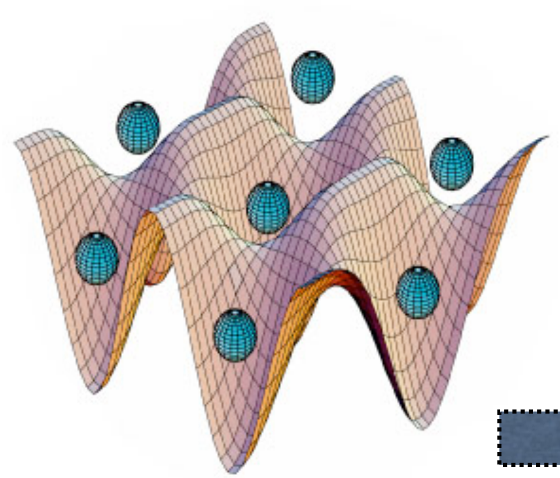


Resistivity?

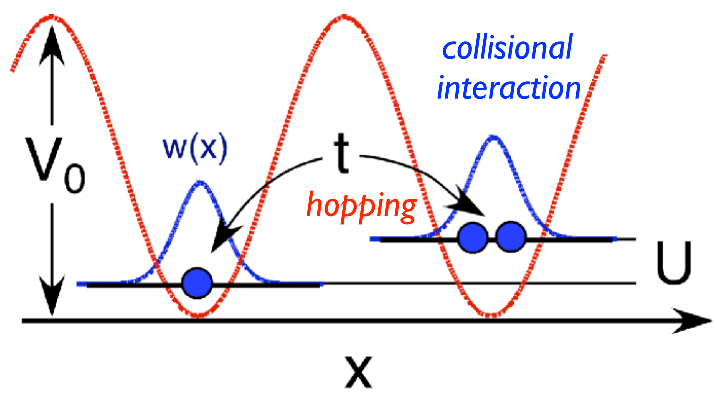
- ~~impurities~~
- ~~phonons~~
- ~~lattice dislocations~~
- **particle scattering**

The Hubbard model

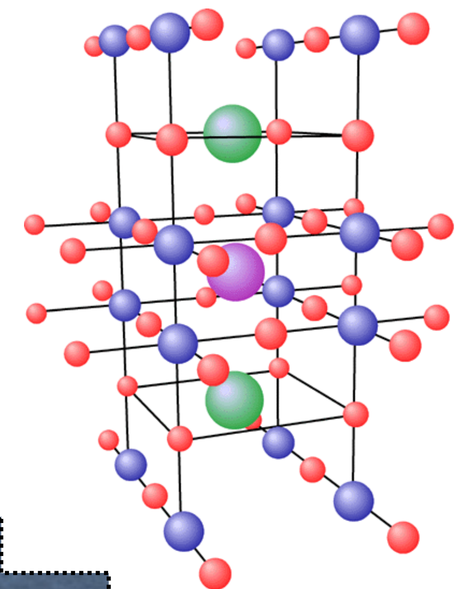
$$\hat{H}_{\mathrm{HM}} = -t \sum_{\langle j,\ell \rangle,\sigma} \hat{c}^{\dagger}_{j\sigma}\hat{c}_{\ell\sigma} + U \sum_{\ell} \hat{n}_{\ell\uparrow}\hat{n}_{\ell\downarrow}$$



Atoms: counter-propagating laser beams produce a sinusoidal potential



Interplay of tunnelling and on-site interaction



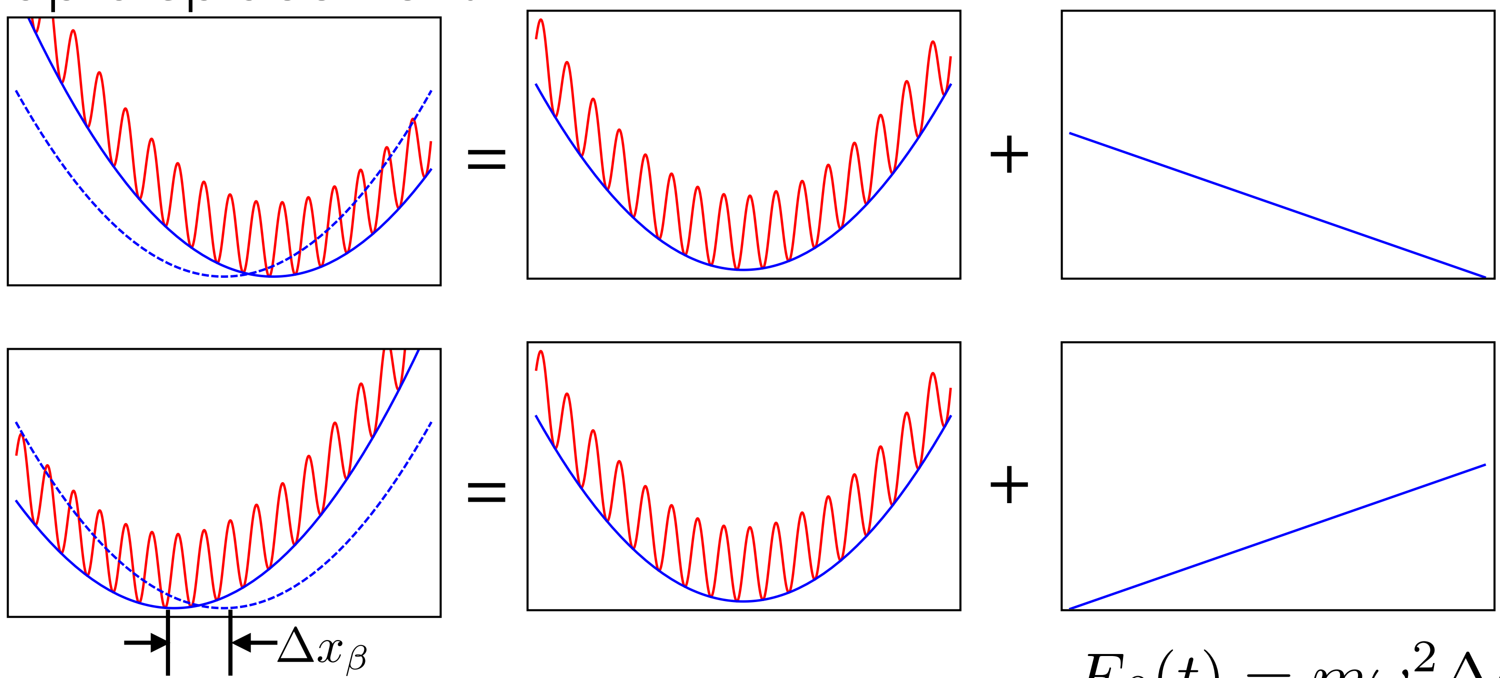
Electrons: ionic lattice

Neglects phonons, dislocations, and impurities, extended w(x), …

AC conductivity of atoms in a lattice

Proposal: Zhigang Wu, E. Taylor, E. Zaremba, *EPL* **110**, 26002 (2015)

periodic trap displacement | static trap | periodic uniform force



(+unimportant offset)

$$F_\beta(t) = m\omega_0^2 \Delta x_\beta \cos[\omega t]$$
$$= F_\beta(\omega) e^{-i\omega t} + \text{c.c.}$$

$$\boxed{F_\beta(\omega) = m\omega_0^2 \Delta x_\beta / 2}$$

Related work:
A. Tokuno and T. Giamarchi, *PRL* **106**, 205301 (2011)
Zhigang Wu and E. Zaremba, *Annals of Physics* **342**, 214 (2014)

time-dependent gauge field

The applied force $F_\beta(\omega) = m\omega_0^2 \Delta x_\beta / 2$

can be seen as a spatially uniform gauge field

$A(\omega) = m\omega_0^2 \Delta x_\beta / 2i\omega$ (so that $F = -\partial_t A = i\omega A$)

which ***writes time-varying phase onto hopping***

$\hat{H}_x = -t_0 \sum_j e^{i\lambda} \hat{c}_j^\dagger \hat{c}_{j+1} + \mathrm{h.c}$ where $\lambda(t) = a_L A(t)/\hbar$

Pierls phase

Linear response: $\lambda \ll 1$ (no Bloch oscillations)

Linear response: *no Bloch oscillations*

Bloch oscillations: wave packet group velocity oscillates at

$$\omega_{\mathrm{B}} = a_L F / \hbar$$

for a static force. For a periodic force, w.p. displacement is

$$\sim \sin\left[(\omega_{\mathrm{B}}/\omega)\sin\omega t\right]$$

But we are looking for a response at the **drive frequency** (and no other frequency). This can only be true for

$\omega_{\mathrm{B}} \ll \omega$ or $F \ll \hbar\omega/a_L$ or $\lambda \ll 1$

-No Bloch oscillations
-No modification of tunnelling in Floquet picture
-Small Pierls phases

Linear response: *no Bloch oscillations*

Bloch osci

s at

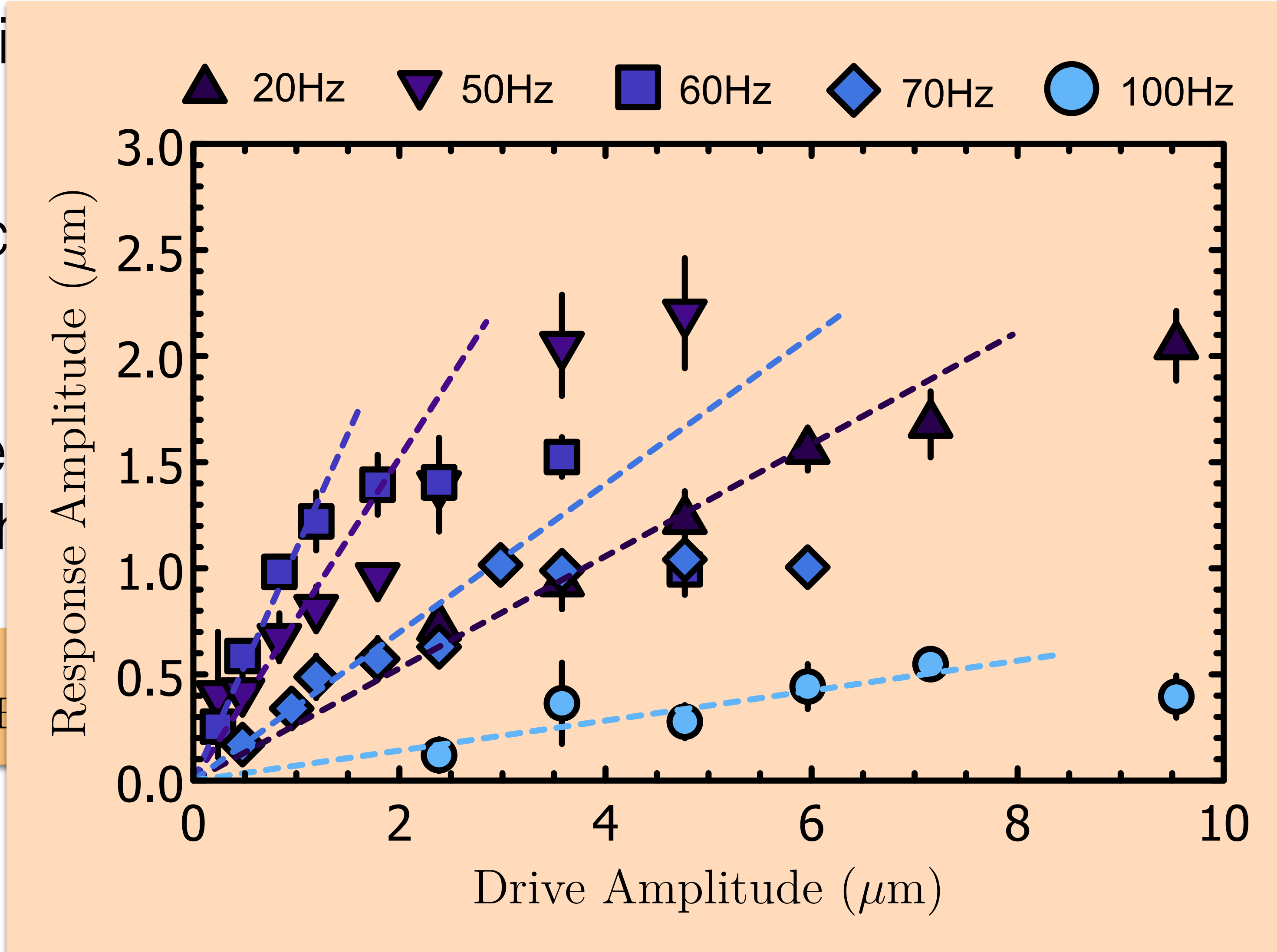
for a static

nt is

But we are

cy

(and no oth



ω_{E}

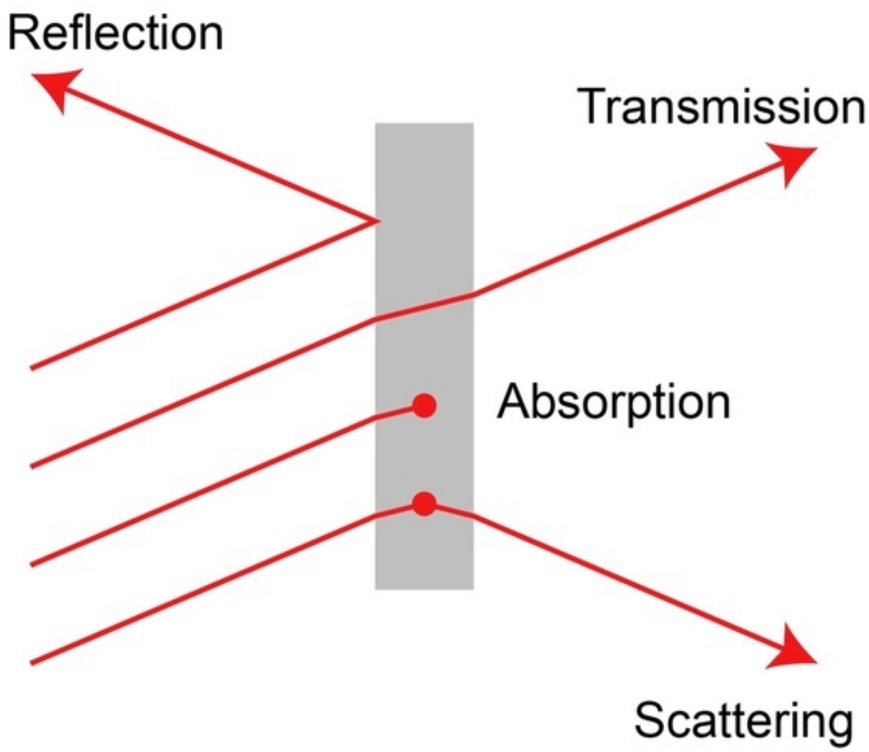
-No

-No

-Small Pierls phases

Optical conductivity

Conductivity without connection to external reservoirs



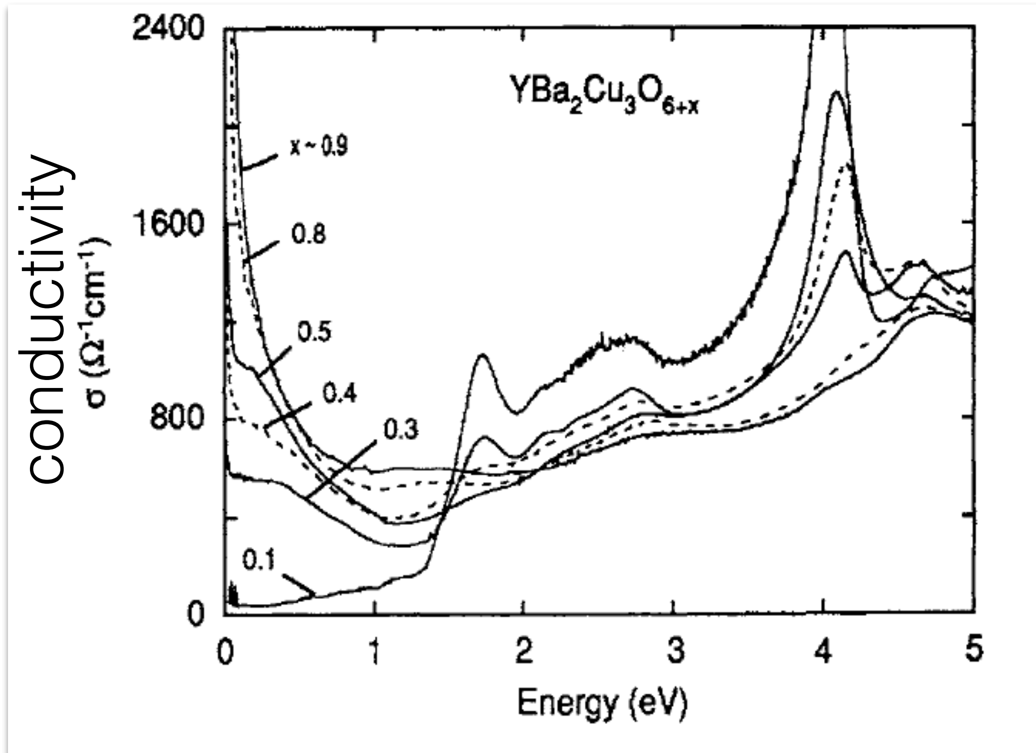


FIG. 10. In-plane ($E \perp c$) optical conductivity $\sigma(\omega)$ obtained from a Kramers-Kronig analysis of the reflectivity data for various compositions of $YBa_2Cu_3O_{6+x}$. Adapted from Cooper, Reznik, *et al.*, 1993.

Relevant frequencies:
1 THz - 10^3 THz

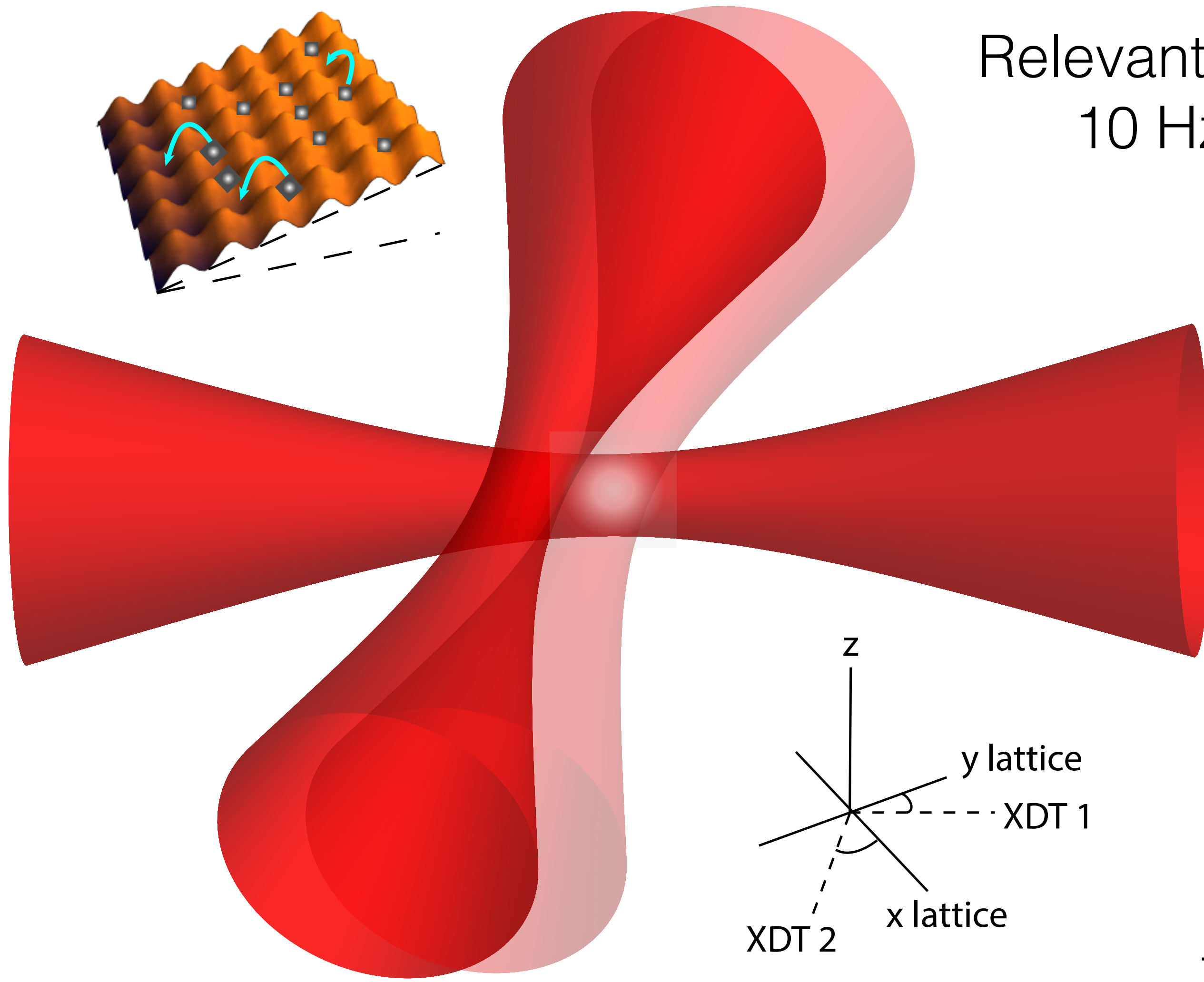
Electrodynamics of high-Tc superconductors, Basov and Timusk, *RMP* (2005) and references therein.

ac (“optical”) conductivity for atoms

Implementation:

1. Apply force with moving optical tweezers

Relevant frequencies:
10 Hz - 10^4 Hz



displace beam with
piezo actuated mirrors

typical force:
0.1 μeV/m

ac ("optical") conductivity for atoms

Implementation:

1. Apply force with moving optical tweezers

2. Measure response with in-situ fluorescence

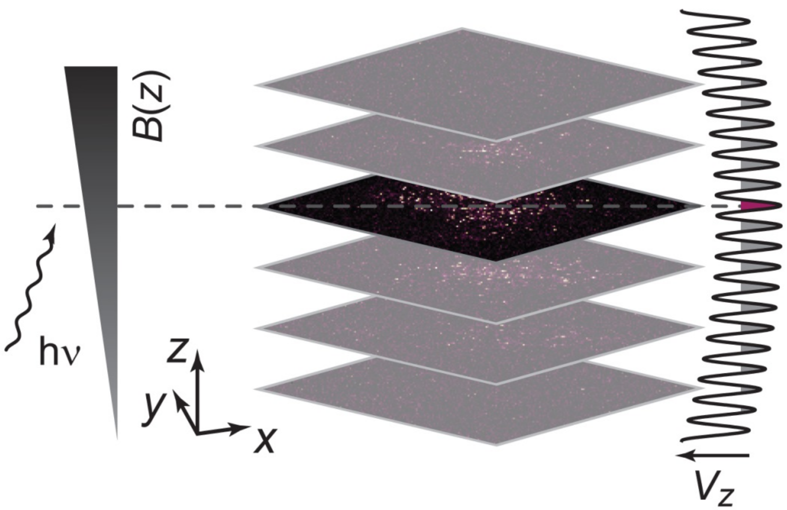
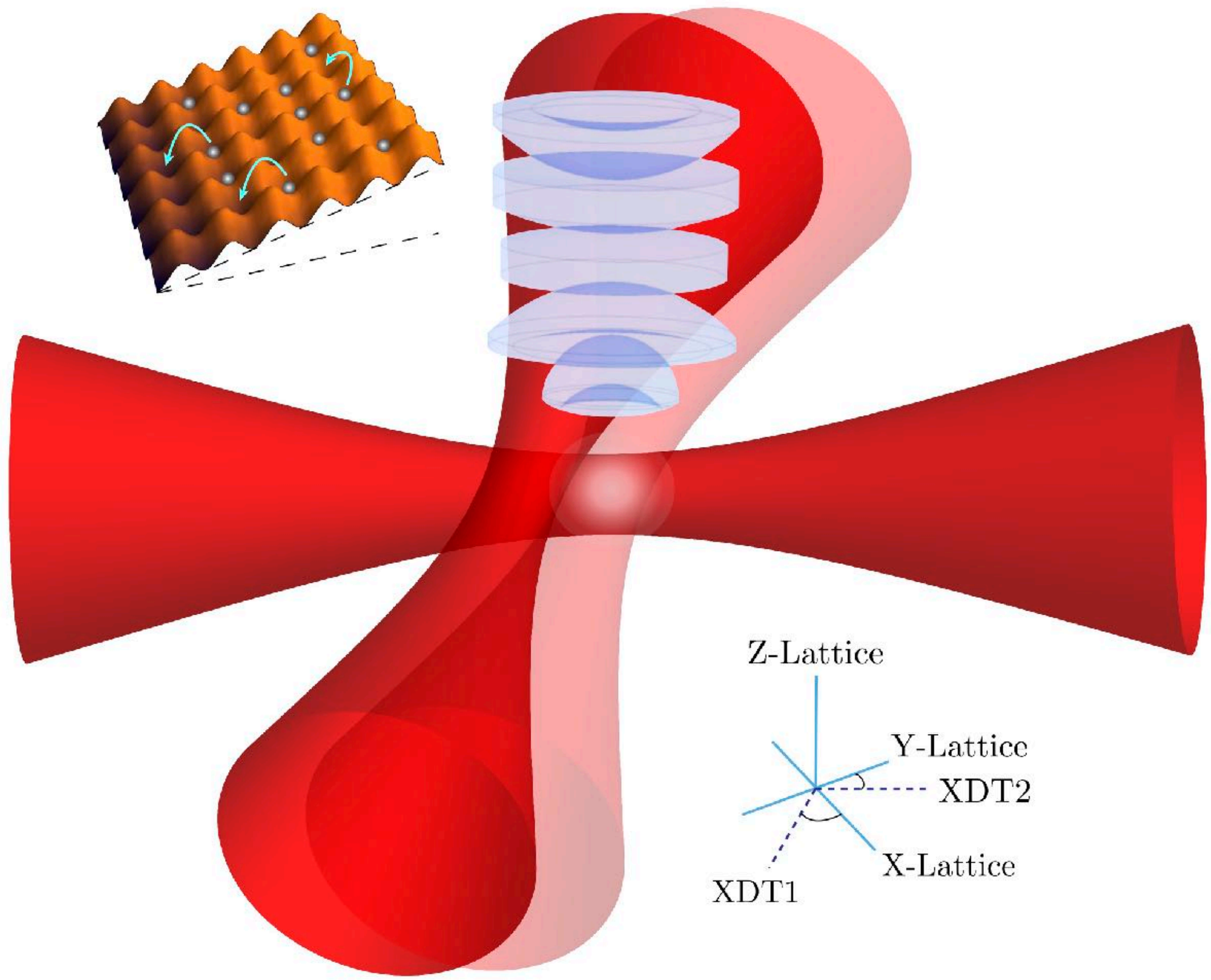


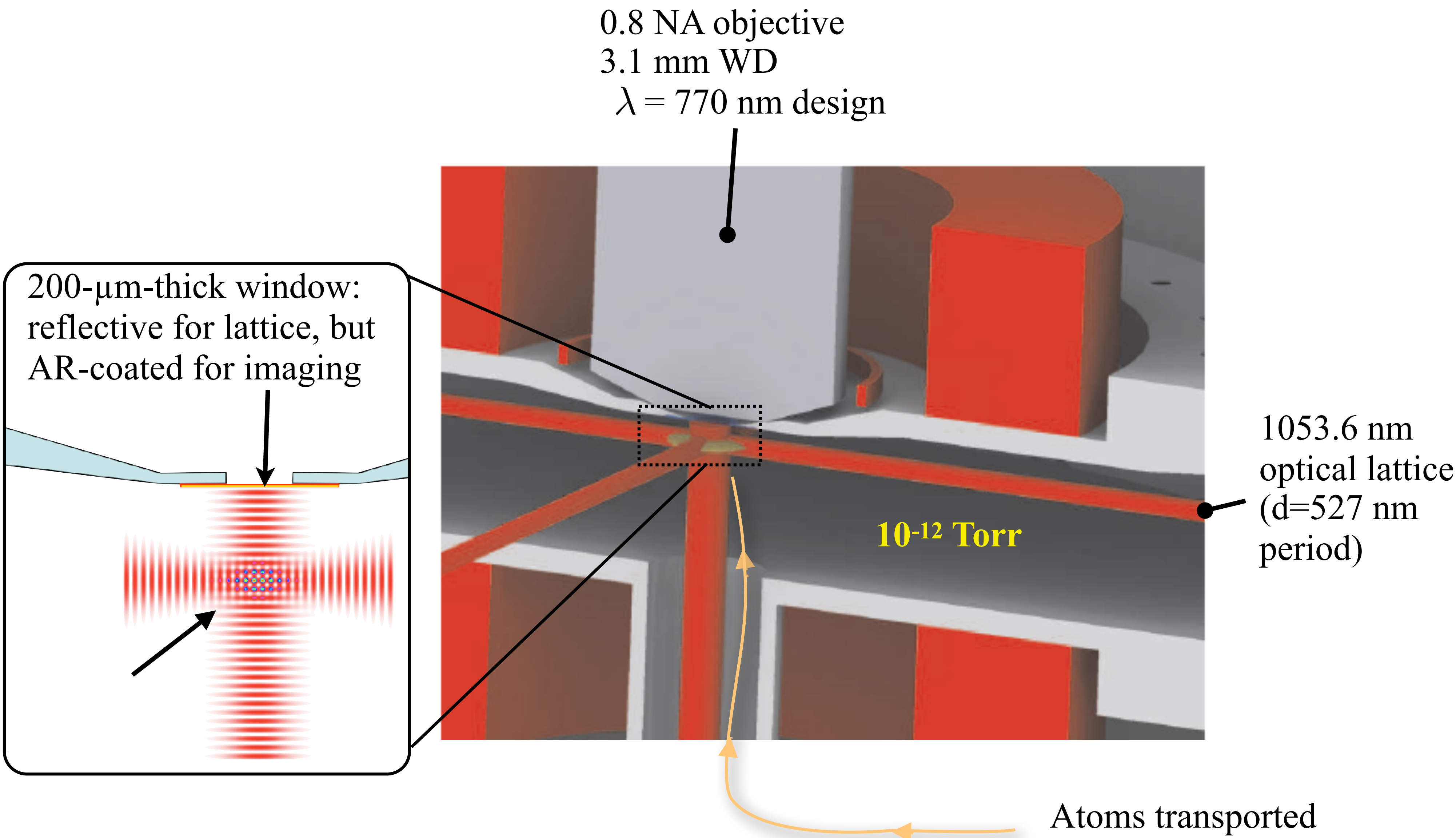
Image 4 central xy planes

Drive and observe in xy plane only.



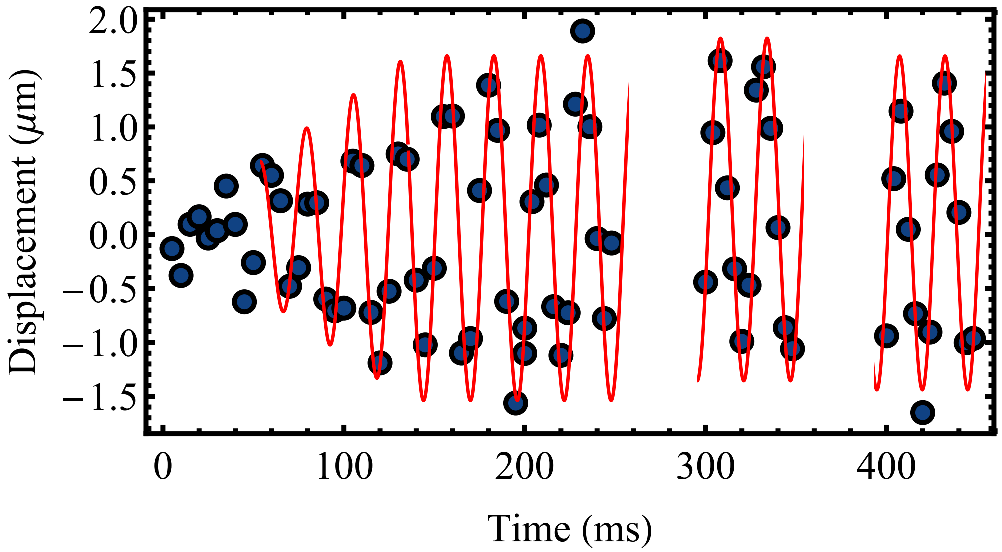
in-situ imaging of atoms in lattices: Chicago, Hamburg, Harvard, Kyoto, MIT, Munich, PSU, Princeton, Strathclyde, Tokyo, Zurich, …

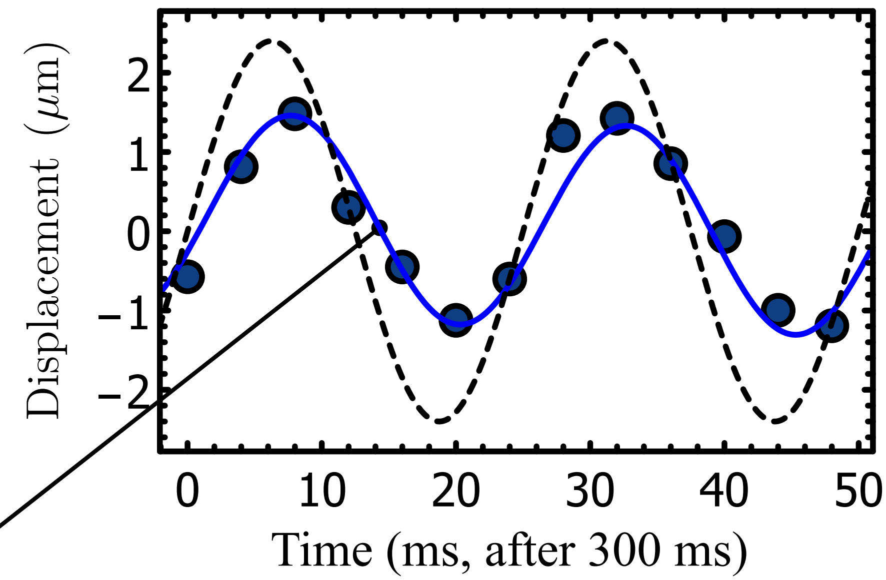
High-resolution in-situ probe



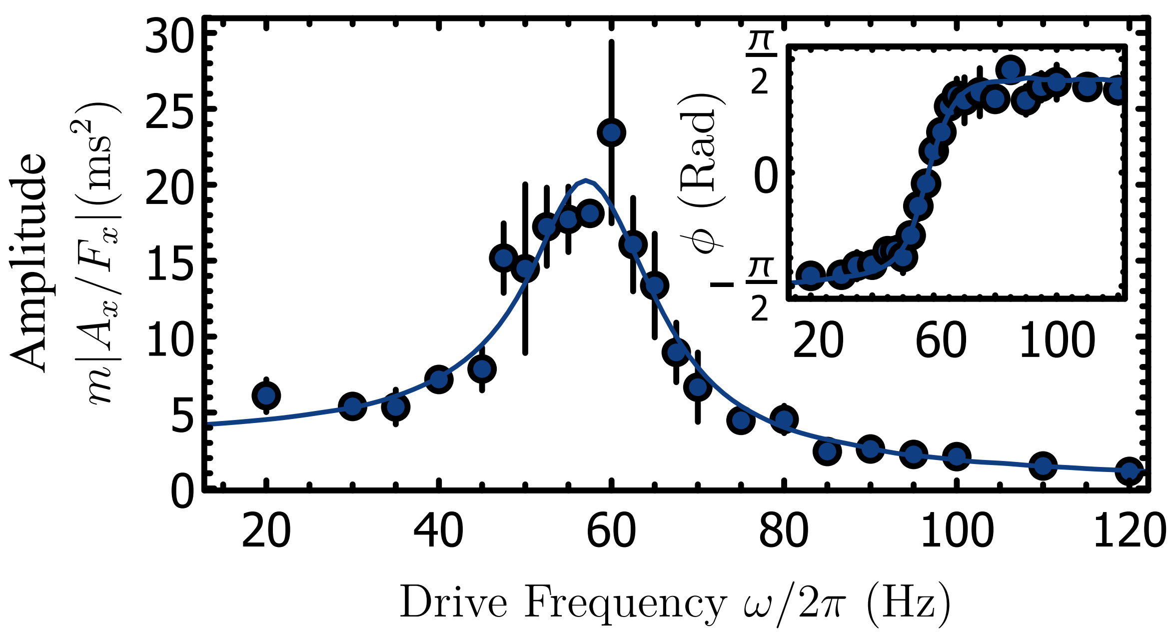
in-situ imaging of atoms in lattices: Chicago, Hamburg, Harvard, Kyoto, MIT, Munich, PSU, Princeton, Strathclyde, Tokyo, Zurich, …

Response: Centre-of-mass displacement



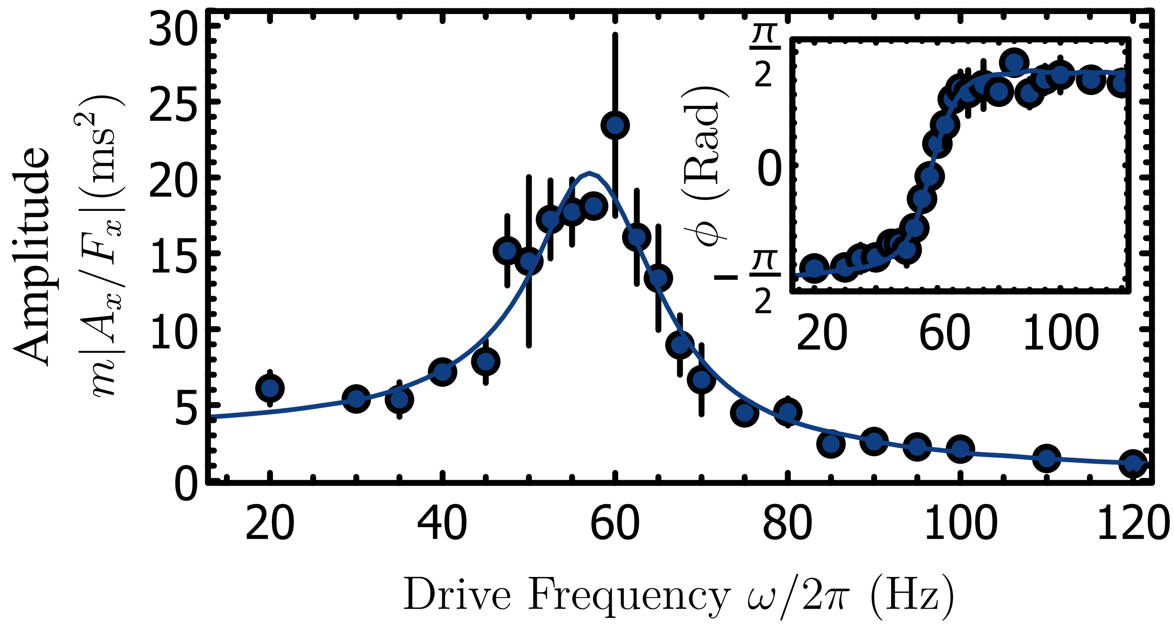


Fit steady-state response to $R_\alpha(t) = A_\alpha \cos\left[\omega t - \phi_\alpha\right]$



Repeat for each drive frequency

Response: Total particle current



c.m. motion reveals the **total current**:

$$\langle \hat{J}_\alpha(t)\rangle = N d\langle \hat{R}_\alpha\rangle/dt$$

For a single-frequency response,

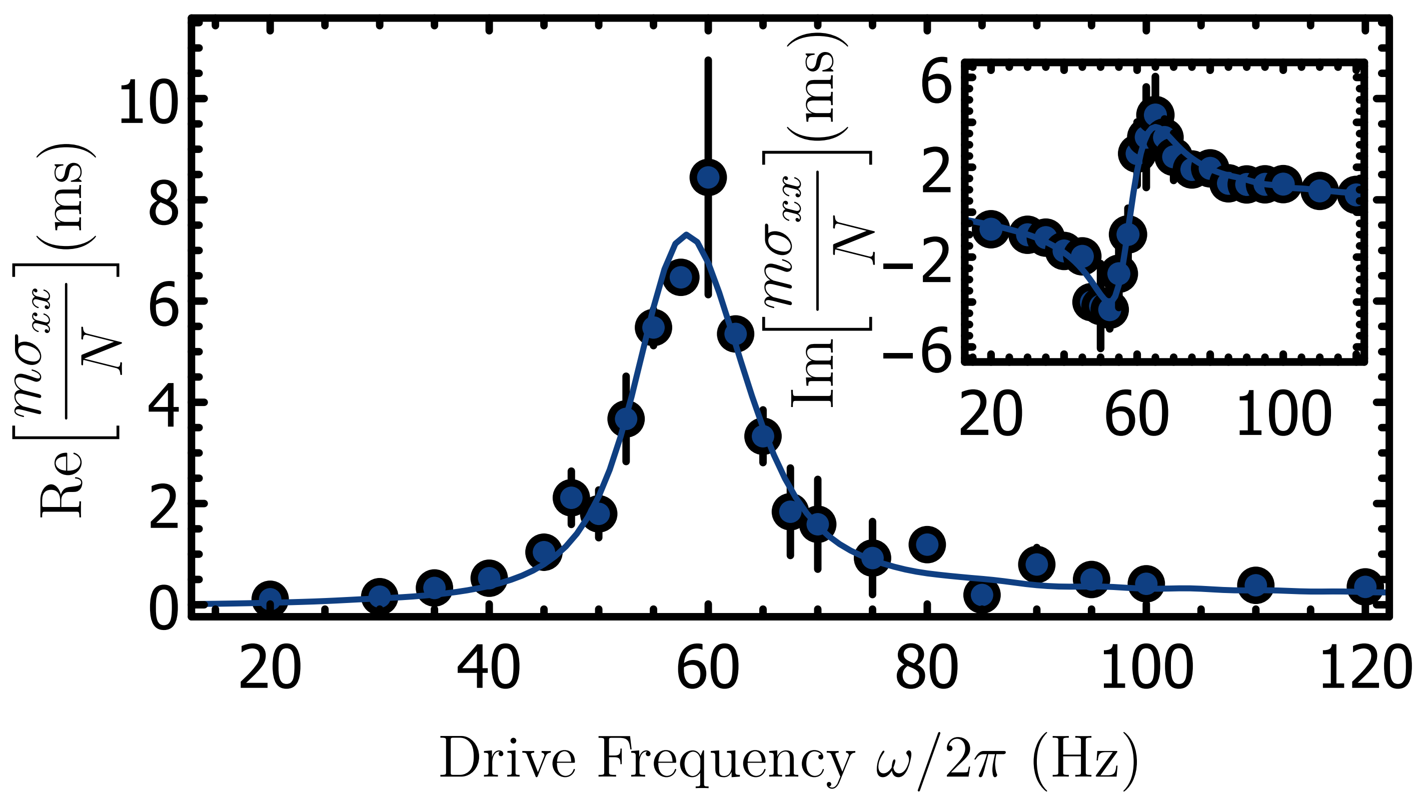
$$\langle \hat{J}_\alpha(t)\rangle = J_\alpha(\omega)e^{-i\omega t} + \mathrm{c.c.}$$

$$J_\alpha(\omega) = \frac{N\omega A_\alpha e^{i\phi_\alpha}}{2i}$$

Now that we have force & current, use Ohm's Law:

$$J_\alpha(\omega) = \sigma_{\alpha\beta}(\omega) F_\beta(\omega)$$

Wu, Taylor, Zaremba, *EPL* (2015)



Conductivity!

-"Optical" conductivity: intrinsically AC technique

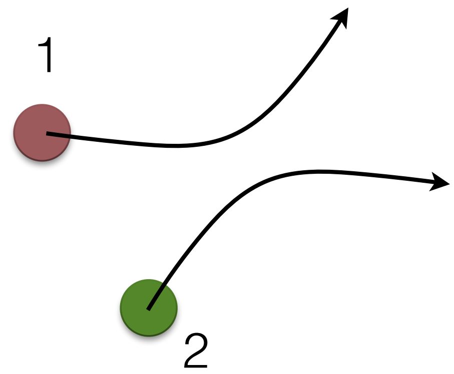
-Global (not local): J is the sum of all currents

-Exact relation (no local density approx, etc.)

-Direct measurement of conductivity (no model requ'd)

current, conductivity, and sum rules

Collisions & Galilean invariance



Total current conserved?

$$\vec{J} = \vec{v}_1 + \vec{v}_2$$

<u>Free space</u>

Total momentum conserved

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

Quadratic dispersion:

$$\vec{v} = \vec{p}/m$$

thus $\vec{J}$ **conserved**.

<u>Lattice</u>

Quasi-momentum conserved

$$\vec{Q} = \vec{q}_1 + \vec{q}_2$$

Tight binding dispersion:

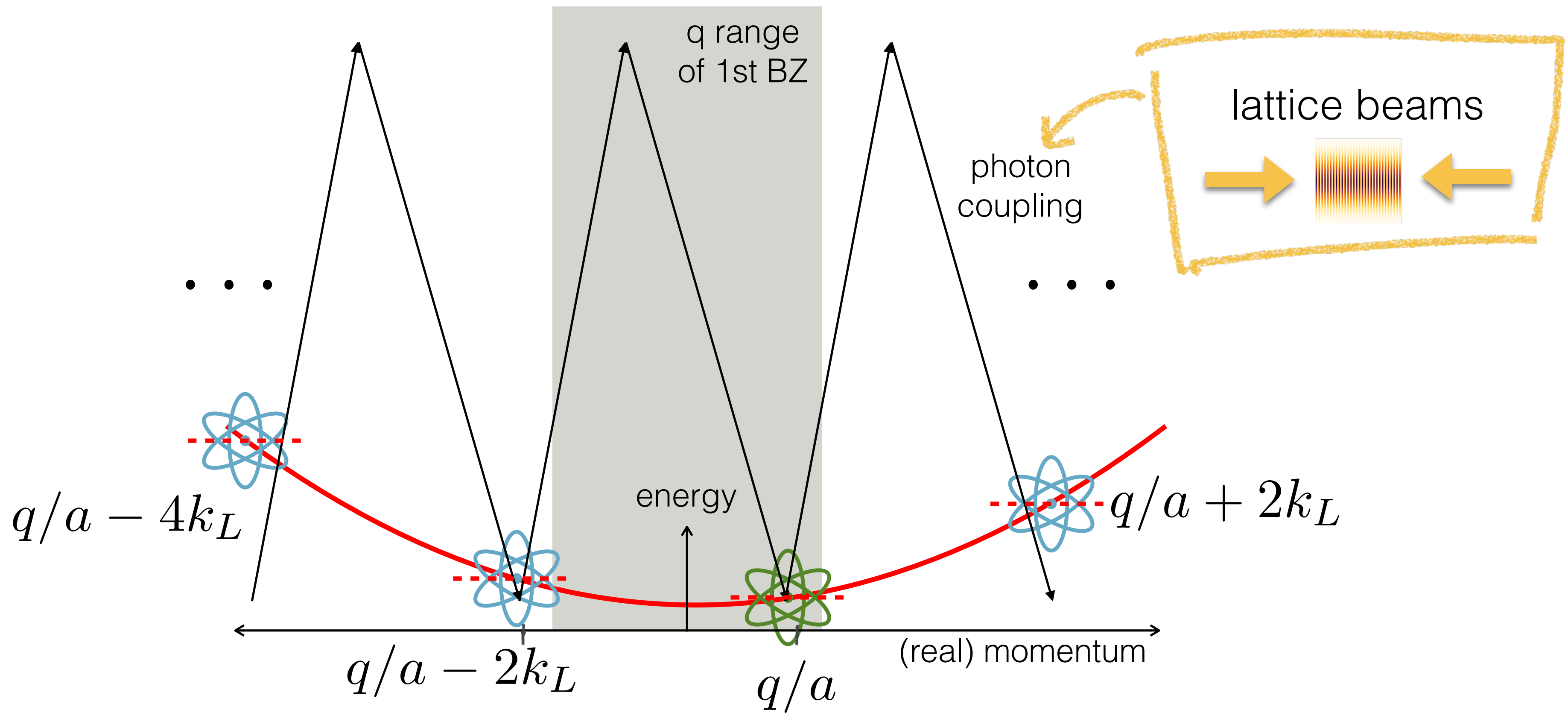
$$v_x = \frac{2t_0 a_L}{\hbar} \sin(q_x a)$$

now $\vec{J}$ **not conserved**.

Dispersion relation is for atom+photon quasiparticles:

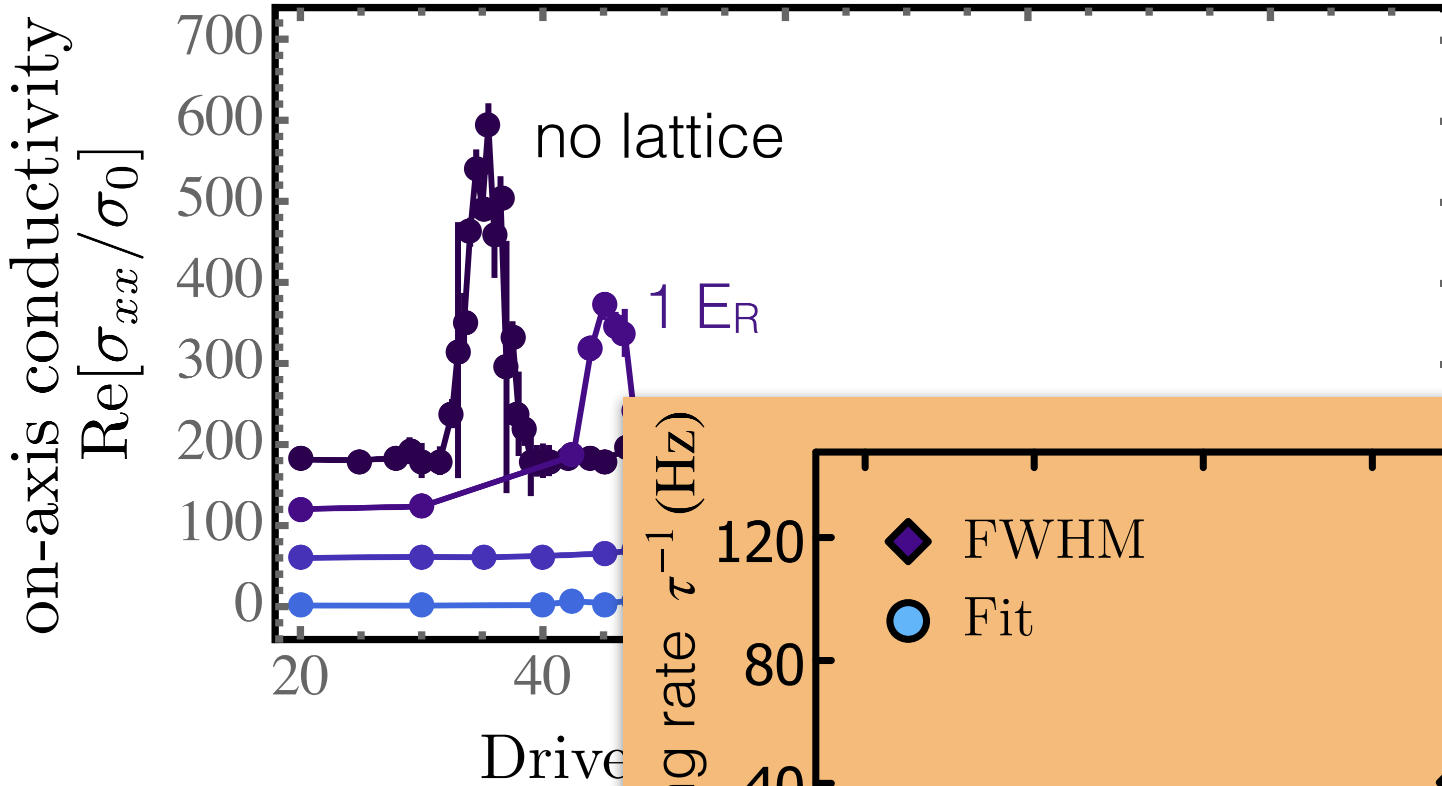
Bloch eigenstate **q** is

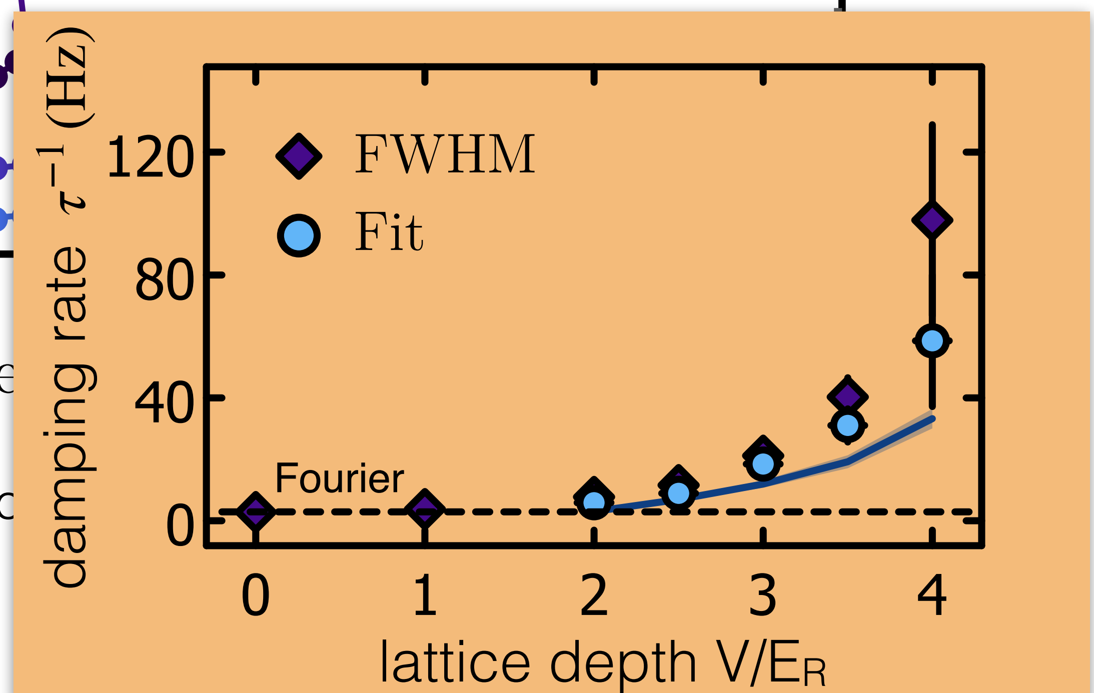
$$|\psi\rangle = c_0|\;q\;\rangle + c_{+1}|\;q+2k_L\;\rangle + c_{-1}|\;q-2k_L\;\rangle + c_{+2}|\;\cdots$$



Conserving total q in a collision does not conserve atomic momentum (and thus not particle current).

Current damping requires breaking of Galilean invariance, accomplished here by the lattice.



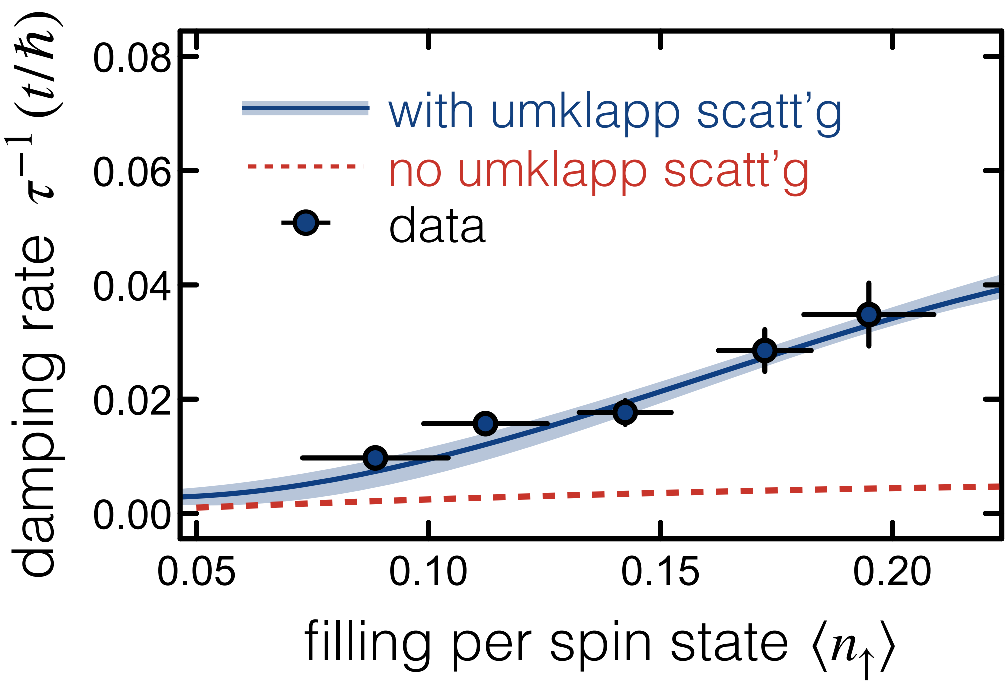


(Resonant peak: trap effec

W. Kohn *Phys. Rev.* **123**, 1242 (1956)

Umklapp scattering

Bragg reflection of colliding pair off lattice



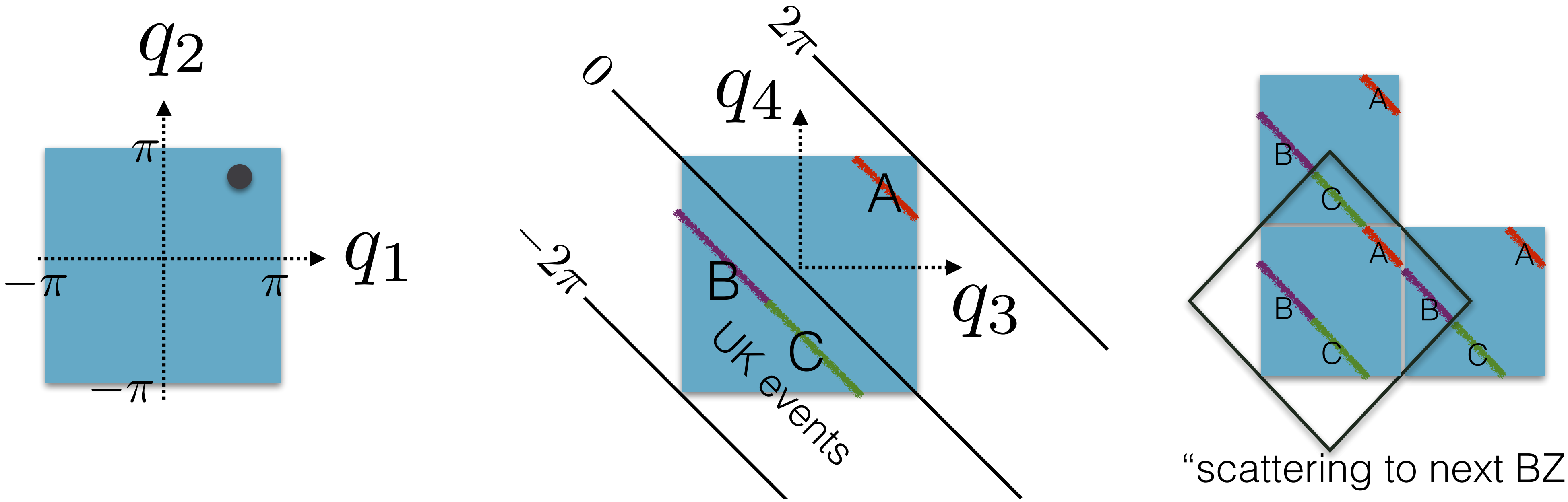
umklapp events:

$$q_1 + q_2 = q_3 + q_4 \pm 2\pi$$

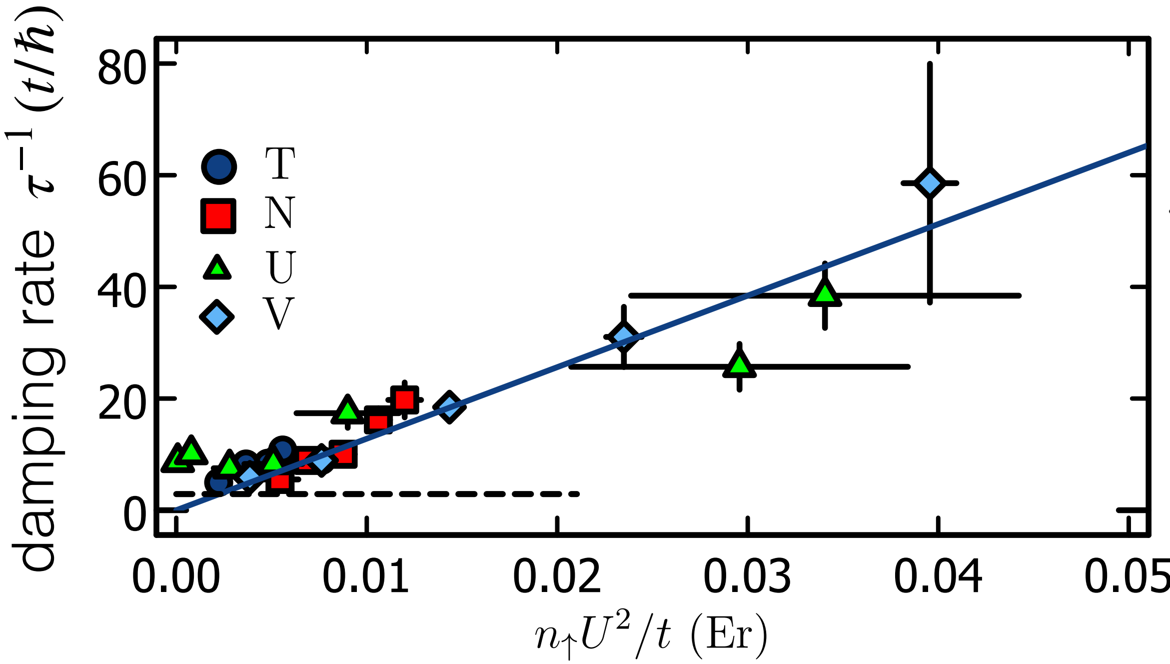
strict q conservation:

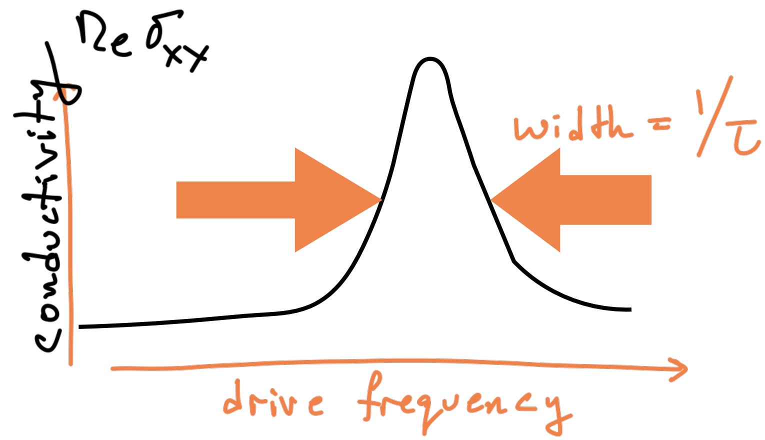
$$q_1 + q_2 = q_3 + q_4$$

[discussion: A. Abrikosov; A. Rosch; …]



Transport time



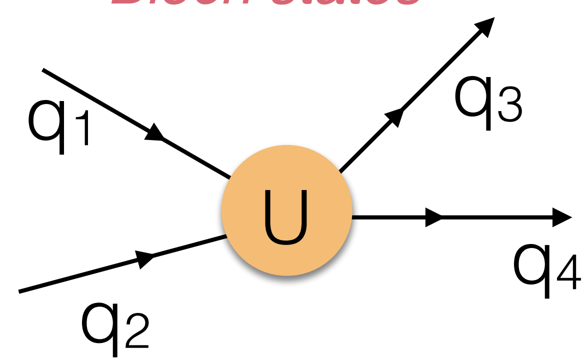


Recall that in quasimomentum basis,

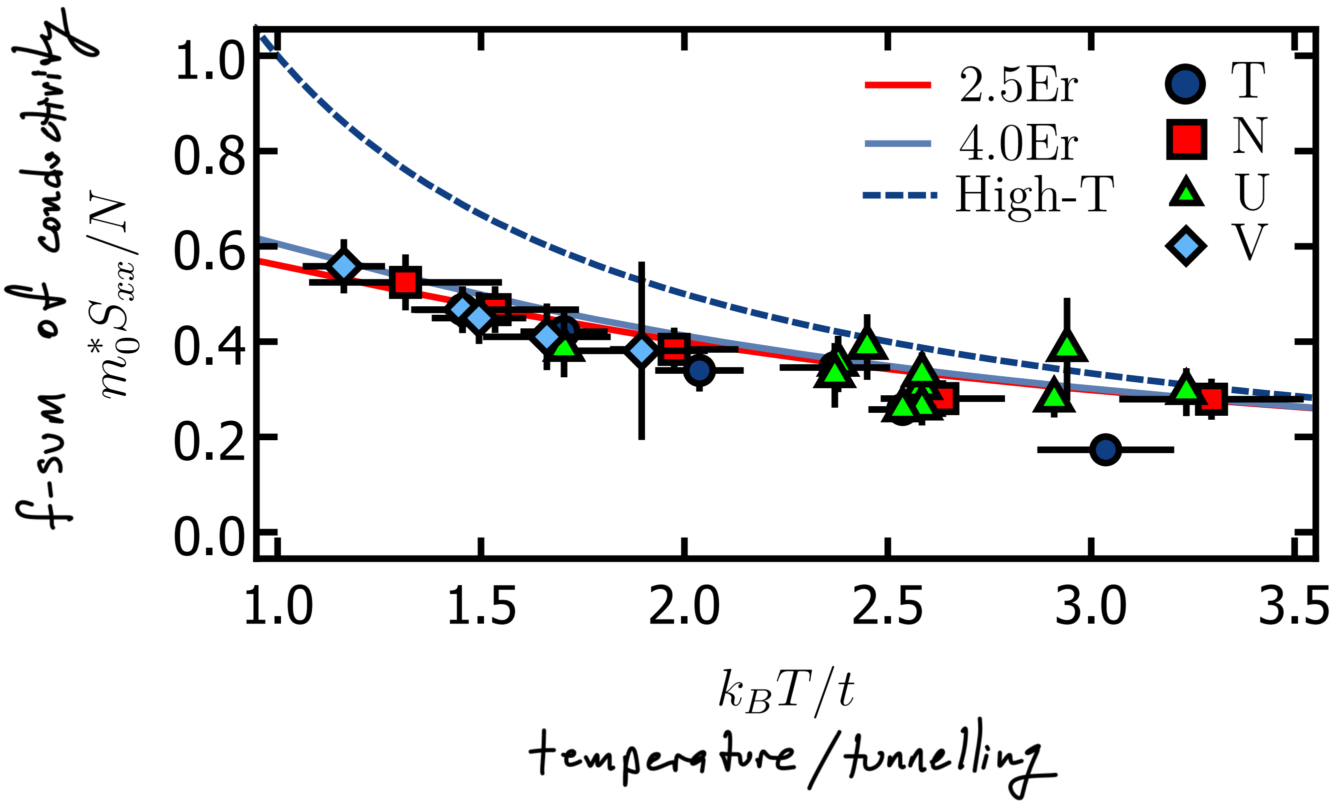
$$\hat{H}_U = \frac{U}{M} \sum_{q_1,q_2,q_3} \hat{c}^{\dagger}_{q_4\uparrow} \hat{c}^{\dagger}_{q_3\downarrow} \hat{c}_{q_2\downarrow} \hat{c}_{q_1\uparrow}$$

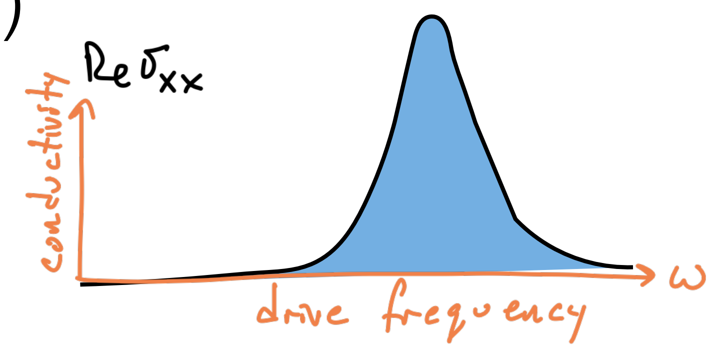
and thus cross-section scales as U^2

scattering between Bloch states



f-sum (kinetic energy)

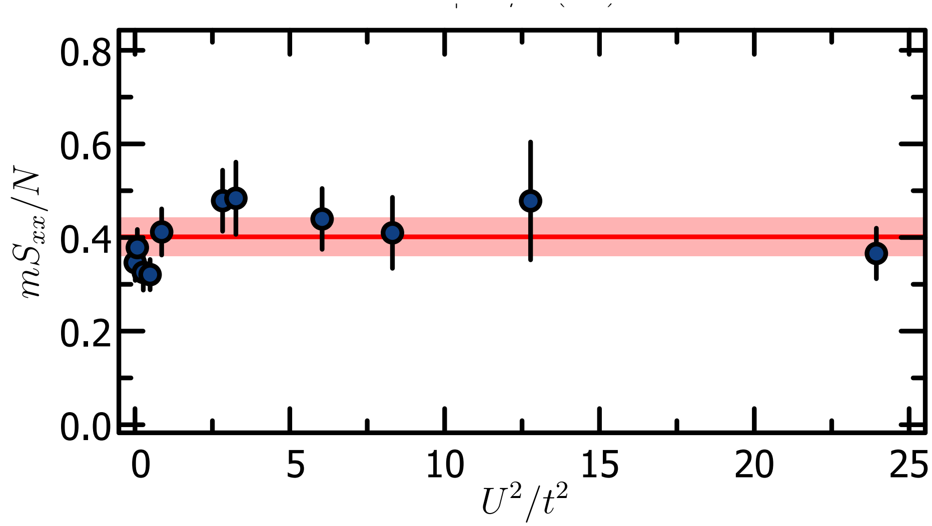




data collapse to KE:

$$\frac{mS_{xx}^{\mathrm{TB}}}{N} = \frac{m}{m_{xx}^*(0)} \frac{I_1(2\beta t_0)}{I_0(2\beta t_0)}$$

independent of U



Hubbard model, TB limit:

$$S_{xx} = -\frac{a_L^2}{\hbar^2}\langle \hat{H}_{0x} \rangle = -\frac{a_L^2}{\hbar^2}\frac{E_K}{d}$$

Summary, Topic 4

- Finite conductivity in a perfect crystal arises from **atom-atom collisions with broken Galilean invariance**.
- **Umklapp** collisions, in which a lattice photon is absorbed, dominates resistivity
- A frequency **sum rule** connects dynamics to thermodynamics, and is independent of trap and interactions
- Frontiers: crossing a phase transition, artificial gauge fields, stronger interactions, …

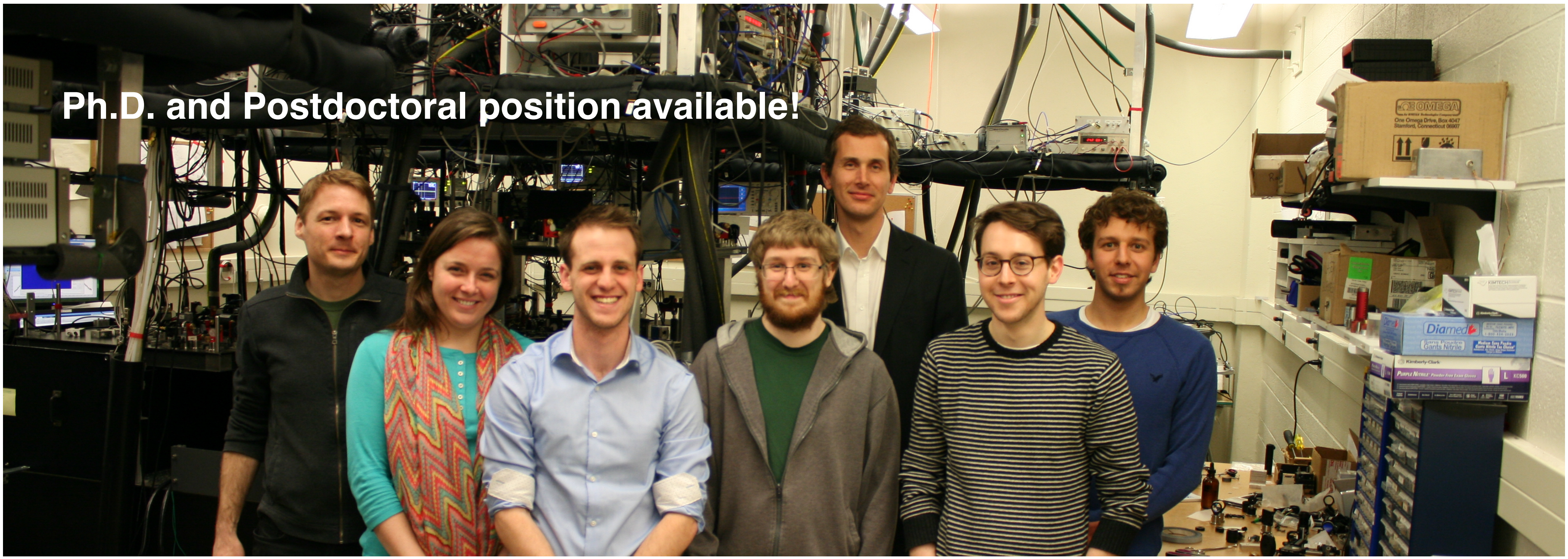
R. Anderson *et al.* arXiv:1712.09965

Research team

Rhys Anderson
Kenneth Jackson
Scott Smale
Matthew Taylor
Vijin Venu
Peihang Xu 许培航

Ben Olsen (-»Yale/NUS)
Fudong Wang 汪福东 (-»南方科技大学)
Stefan Trotzky (-»Metamaterials Halifax)
Dave McKay (-»IBM Watson)
Alma Bardon (-»Morgan Solar)
Scott Beattie (-»NRC Ottawa)
Fabian Böttcher (visit from Stuttgart)

<u>Theory Collaborations</u>
Frédéric Chevy (LKB/ENS)
Tilman Enss (Heidelberg)
Ana Maria Rey (JILA)
Edward Taylor (Toronto)
Zhenhua Yu 俞振华 (中山大学)
Shizhong Zhang 张世忠 (HKU)



Department of Physics
University of Toronto

Canadian Institute
for Advanced Research

NSERC AFOSR ARO

ÉCOLE DE PHYSIQUE
des HOUCHES

Fermions in Optical Lattices

- Introduction
- Fermions, statistics, & exchange
- Matter waves in crystals of light
- The Hubbard model
- Transport

Thank you for your attention and questions!